Optimization of Drinking Water Distribution Costs Using Vogel's Approximation Method (VAM) and Three Modified Methods of VAM (Case Study: Sikumbang Kampar Spring)

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Article Info	Abstract
	Distribution system Springs Sikumbang Kampar is from the primary seller, and the distributor distributes it to the public. The Sikumbang Kampar Springs distributor is experiencing the problem of increasing the cost of distribution because it does not have a distribution pattern. Therefore, it takes a transportation model to complain distribution problems experienced by the distributor of Springs Sikumbang Kampar. The research was conducted to obtain the minimum cost in distributing the drinking water to help distributors solve these problems. For this reason, a method is needed in compiling a mathematical model appropriate to the distribution problem. The methods used in the study are Vogel's Approximation Method (VAM), Improved Vogel's Approximation Method (IVAM), Max-Min Vogel's Approximation Method (MM-VAM), and Modified Vogel's Approximation Method (MVAM). Based on the results of the study, Vogel's Approximation Method generates total cost different initial solutions. Vogel's Approximation Method is more efficient, as it has few iterations to obtain the optimal solution using the stepping stone method. Distributors can consider the use of *Vogel's Approximation Method* in optimizing the distribution costs of drinking water from Sikumbang Kampar Springs. Total distribution transportation costs Sikumbang Kampar Springs per week uses transportation model is Rp 4,580,485.00. This result is more optimal for transportation costs distributor, that is Rp 5.050.000.000,00, so there is a reduction in transportation for Rp 469,515.00.

INTRODUCTION

Transportation methods manage distribution from sources that provide the same product where it is needed (destination) optimally [1]. Product allocation must be regulated because many differences in the allocation cost from a source to a destination. Moreover, differences in the amount of existing transportation capacity and needs at the destination [2]. Transportation problems can be solved by several methods, namely the *Least Cost Method* (LCM)*, North West Corner* (NWC), and *Vogel's Approximation Method* (VAM). VAM usually produces a better initial solution than other methods [2]. VAM is based on the difference between the first most minor transportation cost and the following most negligible cost in a cell in a row or column [3]. VAM also does not require a long time to find the optimal solution [4]. However, in its application, there are still VAM that produce solutions that are not optimal [5]. Several studies have been conducted to improve VAM, including the *Improved Vogel's Approximation Method* (IVAM)*, Max-Min Vogel's Approximation Method* (MM-VAM), and *Modified Vogel's Approximation Method* (MVAM). No theory guarantees that the initial solution is the optimal solution; to obtain the optimal solution, one of them is using the *stepping stone* method [6].

Related research conducted by Shafarda using VAM with Optimal Test *stepping stone* resulted in an initial solution that was not optimal [7]. Subsequent research conducted by Nahar using IVAM on distribution costs by the Logistics Affairs Agency (BULOG) resulted in a more optimal calculation of transportation costs than the company's calculation [8]. Furthermore, the research conducted by Tampubolon, PT. Perkebunan Nusantara IV produced an initial solution using MM-VAM, which was better than LCM [9]. Research in the application of MVAM to minimize transportation costs at the tofu factory conducted by Afiani resulted in a more optimal transportation cost than the factory calculation with a few iterations [10]. Comparing the *revised distribution method* and the *stepping stone* method in the research conducted by Saputri resulted in an optimal solution using the *stepping stone* method, which is better than the *revised distribution method* [11]. Based on the research that has been done, the purpose of this research is to minimize the cost of distributing drinking water from Sikumbang Kampar Springs using the VAM, IVAM, MM-VAM, and MVAM methods and using the *stepping stone* method as an optimization method. VAM usually produces a better initial feasible solution than the NWC and LCM methods [17].

METHOD

The data used in this study is data on the cost of transportation for each means of transportation to *the* destination for the distribution of drinking water from the Sikumbang Kampar Springs each canister. The source of the data used in the study was obtained from an interview with one of the distributors of drinking water from Sikumbang Kampar Springs, precisely in Pulau Sarak Village, Rumbio, on February 24, 2020.

1. Transport Model

Problems in distributing products from sources to several destinations can be arranged into a transportation model. The model is part of a linear programming problem that minimizes transportation costs from the source to the destination area [12]. The transportation model can allocate goods effectively from a source to a destination with the lowest possible cost [13]. The transportation model aims Constraints in transportation problems do not go beyond the source and meet the demands of each destination [14]. In solving transportation problems, transportation problems must be balanced. A transportation problem is said to be balanced if the amount of supply is equal to the number of requests [15]. The steps for solving transportation problems begin by translating the problem into tabular form. Then balance the problem by checking that the quantity of supply equals the quantity of demand. Then, determine the initial feasible solution obtained by four methods: VAM, IVAM, MM-VAM, and MVAM. After the initial solution is obtained, the next step is to test the stepping stone method's optimality.

2. *Vogel's Approximation Method* (VAM)

VAM usually produces a better initial feasible solution than the NWC and LCM methods, although it is more complicated than the two methods [16]. The steps for solving VAM [6] are to calculate the penalty for each row and column obtained based on the difference between the smallest and the second smallest values of the same row and column. Second, choose the row or column with the most significant penalty value. Third, allocate as many products as possible to the cell that has the most negligible cost. Fourth, repeat the first step and stop the process when all allocations have been met. Fifth, calculate the total cost obtained [6].

3. *Improved Vogel's Approximation Method* (IVAM)

IVAM is a method to get an initial feasible solution to transportation problems. IVAM is a modified method of VAM. VAM is modified by using the *Total Opportunity Cost* (TOC) matrix and alternative allocation costs.

The steps of *Improved Vogel's Approximation* begin by creating a matrix (TOC) obtained from the sum of the *Opportunity Cost* (OC) of the row and OC of the column, where for the OC row, it is obtained from the difference of each row with the smallest value of the row [17]. The OC column is obtained from the difference between each column with the smallest value in that column. Next, calculate the *penalty* for each row and column. The *penalty* value is obtained from the difference between the smallest value of the row and column with the second smallest value of the same row and column. Then, select the row and column with the three highest *penalty* costs. After that, the most negligible transportation cost will be chosen from the three highest penalties. These steps are carried out so that all allocations are met. The final step is to calculate the total transportation costs using the initial transportation costs.

4. *Max-Min Vogel's Approximation Method* (MM-VAM)

The MM-VAM method is a modification of VAM. MM-VAM modifies several steps in VAM, namely using a matrix (TOC). The steps of MM-VAM [5] are: First, create a matrix (TOC). Next, calculate the *penalty for* each row and column obtained by subtracting the maximum value from the minimum value for each row and column. Then, pick the two highest *penalties*. After that, the cell with the most negligible transportation cost and the highest allocation will be selected. The process will stop until all products have been allocated and all requests have been fulfilled. The last step is to calculate the results of the initial feasible solution with the initial transportation costs.

5. *Modified Vogel's Approximation* Method (MVAM)

MVAM is one of the modifications of VAM. MVAM produces an initial feasible solution that is more optimal than other methods [18]. The MVAM steps are as follows: First, Subtract the highest transportation cost in each row with each element of each row of the transport table and place it on the top left of the corresponding element. Second, subtract the highest transportation costs in each column with each element of each column of the transportation table and place it on the lower left of the appropriate element. Third, form a reduced matrix whose elements are the sum of the top left and bottom left elements in steps one and two. Then exchange the initial transportation cost with the reduced matrix. Fourth, calculate the penalty by finding the difference between the two most significant numbers in each row and column. Fifth, choose the highest penalty value. If two or more are equal, select the cell with the highest transportation cost (reduced matrix)—sixth, repeating the fourth and fifth steps until all allocations are met. Seventh, calculate the result of the initial feasible solution with the initial transportation cost.

6. *Stepping Stone* Method

According to S. Basriati, the *stepping stone* method is a method that changes the product allocation to get the optimal production allocation using the *trial and error* method [19]. The optimal solution to the transportation problem can be obtained using the *stepping stone* method with the following algorithm [7]: First, perform a closed horizontal or vertical jump alternately by resting on an already filled cell. Second, calculate the cost of the empty cells using a closed path basis with

signs (+) and (-), which are carried out alternately in subsequent cells until they return to the initial empty cell. Third, if all the calculations in the second step are optimistic, then the transportation matrix is optimal. If there is a negative value of the calculation result, then the transportation matrix is not optimal, and the most significant negative value is chosen for the improvement index. Fourth, select the cell with the smallest unit in the jump that is negative. Add the smallest unit to the positive jump, subtract it from the negative jump. Fifth, revise the table by repeating the first step until there are no negative values in evaluating empty cells.

RESULTS AND DISCUSSION
1. Distribution of Sikumbang Kampar Spring Drinking Water
Sikumbang Kampar Spring is used as a drinking water business in Kampar District, Pulau Sarak Village Rumbio. The distributor of drinking water from Sikumbang Kampar Springs is Mr. Zulfikar. He has started his business as a drinking water distributor for Sikumbang Kampar Springs in 2006. The distribution of drinking water from Mr. Zulfikar's Sikumbang Kampar Springs is distributed to five destinations, namely Pekanbaru, Tapung, Tandun, Ujung Batu and Pulau Payung. Mr. Zulfikar uses four means of transportation to distribute drinking water from the Sikumbang Spring. The total demand in a week with a total supply of transportation equipment is not balanced. So the distribution of drinking water from the Sikumbang Spring is repeated to meet all water demands in a week.

2. Completion of the First Distributed Transportation Model
Based on transportation problems and data obtained from the drinking water distribution business of Mr. Zulfikar's Sikumbang Kampar Springs, the number of requests is more significant than a total supply of transportation equipment, resulting in an unbalanced transportation model. For the transportation model to be balanced, a *dummy* supply is added to the supply line. The transportation model can be seen as follows:

Purpose Function:
$$\text{Min } Z = 2.741x_{11} + 2.593x_{12} + 3.185x_{13} + 4.222x_{14} + 1.556x_{15} + 3.188x_{21}$$
$$+ 2.875x_{22} + 3.500x_{23} + 4.125x_{24} + 1.625x_{25} + 3.222x_{31} + 3.000x_{32}$$
$$+ 3.500x_{33} + 4.222x_{34} + 1.556x_{35} + 3.308x_{41} + 2.923x_{42} + 3.846x_{43}$$
$$+ 4.769x_{44} + 2.000x_{45} + 0x_{51} + 0x_{52} + 0x_{53} + 0x_{54} + 0x_{55}$$

Constraint Function:
Paysupply:
$$\sum_{j=1}^{5} x_{1j} \le 135; \sum_{j=1}^{5} x_{2j} \le 80; \sum_{j=1}^{5} x_{3j} \le 90; \sum_{j=1}^{5} x_{4j} \le 65; \sum_{j=1}^{5} x_{5j} \le 1110.$$

Request:
$$\sum_{j=1}^{5} x_{i1} \le 560; \sum_{j=1}^{5} x_{i2} \le 140; \sum_{j=1}^{5} x_{i3} \le 540; \sum_{j=1}^{5} x_{i4} \le 110; \sum_{j=1}^{5} x_{i5} \le 130.$$

a. Completion of the First Distribution Using the VAM Method
The first step, calculate the penalty of each row and column. The penalty value is obtained from the difference between the smallest value of the row and column with the second smallest value of the same row and column, then chooses the most significant penalty value, then allocates

as many products as possible to the cell with the most negligible cost. Here is a table with all the allocations that have been fulfilled using the VAM method:

Table 1. Initial solution using VAM Method for First Distribution

| Transport | Destination Area | | | | | Supply |
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	2741 135	2593	3185	4222	1556	135
Grandmax	3188	2875 40	3500	4125	1625 40	80
L300	3222	3000	3500	4222 90	1556	90
Carry	3308	2923 65	3846	4769	2000	65
Dummy	0 425	0 35	0 540	0 110	0	1110
Demand	560	140	540	110	130	1480

After all, allocations are met, the initial solution has been obtained with the total cost of the initial solution generated using the VAM method is $Z = 880.070$.

b. Completion of the First Distribution Using the IVAM Method

The first step in solving transportation problems using IVAM is to create a TOC matrix obtained from the sum of the *Opportunity Cost* (OC) rows and OC columns, where for row OC, it is obtained from the difference between each row and the smallest value of the row. The OC column is obtained from the difference in the value of each column with the smallest value. The following is the column TOC table for the transportation problem of the Sikumbang Kampar Spring drinking water distribution business:

Table 2. TOC Matrix of Transportation Costs

| Transport | Destination Area | | | | | Supply |
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	1185	1037	1629	2763	0	135
Grandmax	2010	1532	2190	2500	69	80
L300	2147	1851	2259	2763	0	90
Carry	1875	1253	2507	3413	444	65
Dummy	0	0	0	0	0	1110
Demand	560	140	540	110	130	1480

After obtaining the TOC matrix, the next step is to calculate the penalty for each row and column. The penalty value is obtained from the difference between the smallest value of the row

and column with the second smallest value of the same row and column, then selecting the row and column with the three highest penalty costs. Then choose the most negligible transportation cost from the three highest penalties. The following is a table with all allocations that have been fulfilled using the IVAM method:

Table 3. Initial solution using the First Distribution IVAM Method

Transport	Destination Area					Supply
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	2741 5	2593	3185	4222	1556 130	135
Grandmax	3188 5	2875 75	3500	4125	1625	80
L300	3222 90	3000	3500	4222	1556	90
Carry	3308	2923 65	3846	4769	2000	65
Dummy	0 460	0	0 540	0 110	0	1110
Demand	560	140	540	110	130	1480

After all, allocations are met, the initial solution using the IVAM method has been obtained with a total initial solution cost of $Z = 927.525$.

c. Completion of the First Distribution using the MM-VAM Method

First, create a TOC matrix. After the TOC matrix is obtained, then calculate the penalty for each row and column. The penalty value is obtained from subtracting the maximum value with the minimum value for each row and column. Then, pick the two highest penalties. Next, select the cell that has the most negligible transportation cost. Here is a table with all the allocations that have been fulfilled using the MM-VAM method:

Table 4. Initial solution using the First Distribution MM-VAM Method

Transport	Destination Area					Supply
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	2741	2593 5	3185	4222	1556 130	135
Grandmax	3188 10	2875 70	3500	4125	1625	80
L300	3222 90	3000	3500	4222	1556	90
Carry	3308	2923 65	3846	4769	2000	65
Dummy	0 460	0	0 540	0 110	0	1110
Demand	560	140	540	110	130	1480

After all, allocations are met, the total cost of the initial solution generated using the MM-VAM method is $Z = 928,350$.

d. Completion of the First Distribution Using the MM-VAM Metode Method

The first step, subtract the highest transportation costs in each row with each element of each row of the transportation table and place it on the top left of the corresponding element. Then subtract the highest transportation cost in each column by each element of each column of the transportation table and place it on the lower left of the appropriate element. Next, form a reduced matrix whose elements are the sum of the top left and bottom left elements in steps one and two. Then exchange the initial transportation cost with the reduced matrix.

Table 5. Matrix of Reduced Transportation Costs for the MVAM. Method

Transport	Destination Area					Supply
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	2048	2036	1698	547	3110	135
Grandmax	1057	1375	971	644	2875	80
L300	1086	1222	1068	547	3110	90
Carry	1461	1923	923	0	2769	65
Dummy	0	0	0	0	0	1110
Demand	560	140	540	110	130	1480

After the reduced matrix is obtained, calculate the penalty by finding the difference between the most significant numbers in each row and column. After choosing the highest penalty value, after all, allocations are fulfilled, enter all cell allocations from the reduced matrix to the transport table, then calculate the result of the initial feasible solution.

Table 6. Initial Solution Using the First Distribution MVAM Method

Transport	Destination Area					Supply
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	2741	2593	3185 (135)	4222	1556	135
Grandmax	3188	2875	3500	4125 (40)	1625 (40)	80
L300	3222	3000	3500	4222	1556 (90)	90
Carry	3308	2923 (65)	3846	4769	2000	65
Dummy	0 (560)	0 (75)	0 (405)	0 (70)	0	1110
Demand	560	140	540	110	130	1480

After all, allocations are met, the initial solution has been obtained, with the total cost of the initial solution generated using the MVAM method, namely Z=990.010.

e. The First Distribution Optimal Solution Using the *Stepping Stone* Method

The first distribution optimal solution for the initial solution using the VAM, IVAM, MM-VAM, and MVAM methods produces the exact total optimal solution cost and allocation. However, to get the optimal solution, the VAM method obtains the optimal solution with two iterations, the IVAM method, MM-VAM, and MVAM with four iterations using the *stepping stone* method. The optimal solution for the first distribution can be seen in the following table:

Table 7. Results of the First Distribution Optimal Solution Using the *Stepping Stone* Method

Transport	Destination Area					Supply
	Pekanbaru	Tapung	Tandun	Ujung Batu	Pulau Payung	
Engkel	2741 100	2593 35	3185	4222	1556	135
Grandmax	3188	2875 40	3500	4125	1625 40	80
L300	3222	3000	3500	4222	1556 90	90
Carry	3308	2923 65	3846	4769	2000	65
Dummy	0 460	0	0 540	0 110	0	1110
Demand	560	140	540	110	130	1480

The optimal solution for the first drinking water distribution from Sikumbang Kampar Springs using the optimal *stepping stone* test method is Rp 874,890,00. Engkel is distributing 100 canisters to Pekanbaru and 35 canisters to Tapung, Grandmax is distributing 40 canisters to Tapung and 40 canisters to Payung Island, L300 is distributing 90 canisters to Payung Island and Carry is distributing water as much as 65 canisters to Tapung.

3. Completion of the Second Distribution Transport Model

After obtaining the optimal solution with the existing allocation, not all requests have been fulfilled. Then it is done again to find the optimal solution for the second distribution, based on the optimal table obtained from the first distribution of the remaining unmet demand for Pekanbaru 460 canisters, Tandun 540 canisters, and Ujung Batu as many as 110 canisters with the same supply of means of transportation. Based on the remaining requests, the VAM method's initial solution is Rp 1,180,055, IVAM Rp 1,130,075, MM-VAM Rp 1,130,075, and MVAM Rp 1,215,035.

After the initial solution is obtained, then the optimality test is carried out. The test will be carried out using the *stepping stone* method. The results of the second distribution optimization for the initial solution using the VAM, IVAM, MM-VAM, and MVAM methods produce the exact total optimal solution cost and allocation, but to get the optimal solution, the VAM and MVAM

methods obtain the optimal solution with three iterations, the IVAM and MM-VAM methods with one iteration using the *stepping stone* method. The optimal solution for the second distribution can be seen in the following table:

Table 8. The Second Distribution Optimal Solution using the *Stepping Stone* Method

Transport	Destination Area			Supply
	Pekanbaru	Tandun	Ujung Batu	
Engkel	2741 / 135	3185	4222	135
Grandmax	3188 / 80	3500	4125	80
L300	3222 / 90	3500	4222	90
Carry	3308 / 65	3846	4769	65
Dummy	0 / 90	0 / 540	0 / 110	740
Demand	90	540	110	1110

The optimal solution for the distribution of drinking water from the Sikumbang Kampar Springs uses the optimal *stepping stone* test method to distribute the two drinking water from the Sikumbang Kampar Springs resulting in a total of Rp 1,130,075.00. Engkel is distributing 135 canisters to Pekanbaru. Grandmax is distributing 80 canisters to Pekanbaru, L300 is distributing 90 canisters to Pekanbaru and Carry is distributing 65 canisters to Pekanbaru.

4. Completion of the Third Distribution Transport Model

After obtaining the optimal solution in the second distribution, not all requests were met. It was carried out again looking for the optimal solution for the distribution of the three drinking water, based on the optimal table obtained from the distribution of the two remaining unfulfilled requests for Pekanbaru 90 canisters, Tandun 540 canisters, and Ujung Batu as many as 110 canisters and with the same supply of means of transportation. Based on the remaining demand, the initial solution for the third distribution using the VAM method is Rp 1,228,895, IVAM Rp.1,235.005, MM-VAM Rp 1,285,005, and MVAM Rp 1,232,195.

After the initial solution is obtained, the optimization test is carried out using the *stepping stone* method. The results of the third distribution optimization for the initial solution using the VAM, IVAM, MM-VAM, and MVAM methods produce the exact total initial solution cost and allocation but to get the optimal solution. The VAM method obtains the optimal solution with one iteration, the IVAM and MM-VAM methods with two iterations, and the MM-VAM method obtains the optimal solution with three iterations of a *stepping stone*. The optimal solution for the first distribution can be seen in the following table:

Table 9. The Third Distribution Optimal Solution Using the *Stepping Stone* Method

Transport	Destination Area			Supply
	Pekanbaru	Tandun	Ujung Batu	
Engkel	2741 25	3185 110	4222	135
Grandmax	3188	3500 80	4125	80
L300	3222	3500 90	4222	90
Carry	3308 65	3846	4769	65
Dummy	0	0 260	0 110	340
Demand	90	540	110	740

The optimal solution for the distribution of drinking water from Sikumbang Kampar Springs uses the optimal *stepping stone* test method to distribute the three drinking water from Sikumbang Kampar Springs, resulting in a total of Rp 1,228,895.00. Engkel is distributing 25 canisters to Pekanbaru and 110 to Tandun. Grandmax is distributing 80 canisters to Tandun, L300 is distributing 90 canisters to Tandun, and Carry is distributing 65 canisters to Pekanbaru.

5. Completion of the Fourth Distribution Transport Model

After obtaining the optimal solution for the third distribution, not all requests were met. It was carried out again looking for the optimal solution for the fourth distribution of drinking water, based on the optimal table obtained from the distribution of the three remaining unfulfilled requests for Tandun 260 canisters and Ujung Batu as many as 110 canisters and still with the same means of transport. Based on the remaining requests, the initial solution for the fourth distribution using the VAM method is Rp 1,346,625, IVAM Rp 1,367,450, MM-VAM Rp 1.352.655, and MVAM Rp 1,346,625.

After the initial solution is obtained, the optimization test is carried out using the *stepping stone* method. The results of the first distribution optimization for the initial solution using the VAM, IVAM, MM-VAM, and MVAM methods produce the exact total initial solution cost and allocation. However, to get the optimal solution of the VAM and MVAM methods with one iteration, the IVAM method obtains the optimal solution with four iterations, and the MM-VAM method obtains the optimal solution with two iterations using the *stepping stone* method. The optimal solution for the fourth distribution can be seen in the following table:

Table 10. The Fourth Distribution Optimal Solution using the *Stepping Stone* Method

Transport	Destination Area		Supply
	Tandun	Ujung Batu	
Engkel	3185 135	4222	135
Grandmax	3500	4125 80	80
L300	3500 60	4222 30	90
Carry	3846 65	4769	65
Demand	260	110	740

The optimal solution for the distribution of drinking water from the Sikumbang Kampar Springs using the optimal *stepping stone* test method for distributing the four-drinking water from the Sikumbang Kampar Springs resulted in a total of Rp 1,346,625.00 Engkel is distributing 135 canisters to Tandun. Grandmax distributes 80 canisters to Ujung Batu, L300 distributes 60 canisters to Tandun and 30 canisters to Ujung Batu, and Carry distributes 65 canisters to Tandun. After obtaining the optimal solution for the fourth distribution, all requests have been fulfilled, then the search for the optimal solution for the distribution of drinking water from Sikumbang Kampar Springs is stopped.

CONCLUSIONS AND SUGGESTIONS

The optimal transportation cost for drinking water distribution from the Sikumbang Spring in one week using the transportation model is Rp 4,580,485.00. This result is more effective than the distributor's calculation before using the transportation model, Rp 5,050,000.00. Based on the research results, the VAM method is more efficient than the IVAM, MM-VAM, and MVAM methods because it has the fewest iterations to obtain the optimal solution using the *Stepping Stone* method.

Based on the research results that have been done, the authors hope that distributors can consider the use of *Vogel's Approximation Method* in optimizing the distribution costs of drinking water from Sikumbang Kampar Springs. Optimization of transportation costs using this method is more efficient and can result in greater profits for distributors. It is recommended that readers use other transportation methods to solve transportation problems to obtain a more efficient method.

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