



Profile of Pre-Service Mathematics Teacher's Algebraic Thinking Based on Systematic-Intuitive Cognitive Style

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Abstract

Algebraic thinking plays an essential role in increasing mathematics problem-solving abilities. In this study, the pre-service mathematics teacher's (PMT's) ability of algebraic thinking is explored based on a systematic-intuitive cognitive style. This study aims to reveal students' algebraic thinking abilities regarding systematic-intuitive cognitive style. Three components of algebraic thinking were analyzed: arithmetic generalization, functional thinking, and generalization and justification. The research approach is qualitative with a case study method. The subjects were 31 PMT's at one of the private universities in Surakarta District, Central Java, Indonesia. Data collection methods were algebraic thinking tests, the Cognitive Style Inventory (CSI) questionnaires, and interview protocol. Four subjects, two PMT's for each cognitive style category were interviewed to reveal their algebraic thinking abilities. The results showed that all subjects were able to solve the functional thinking problem correctly. However, for the generalization arithmetics and justification problems, the PMT's abilities are varied. In addition, the finding also showed the different strategies of systematic and intuitive subjects in solving the problems related to algebraic thinking components. PMT's with a systematic cognitive style solve problems systematically and represent the pattern verbally in the form of a table or word, whereas the PMT's with an intuitive cognitive style solve the problem briefly and visually using pictures pattern. In conclusion, there is a relationship between algebraic thinking ability and cognitive style.

INTRODUCTION

Algebraic thinking is the ability to collect data from various contexts, represent it in words, diagrams, graphs, tables, or equations, and then interpret the representation. Algebraic thinking covers numbers concepts, calculation, symbolization, solving problems, and investigating patterns and functions [1]. Algebraic thinking activity is the activity to develop algebraic thinking skills that involve generalization, abstraction, dynamics thinking, analysis, modeling, and organization [2]. Algebraic thinking is also a reasoning process with generalizing, developing concepts, and formulating methods [3].

Algebraic thinking has two parts: learning to think math and studying the basics of algebra. Algebraic thinking also combines operating at the transformation, generalization, and global meta-level [1]. Transformation involves factoring, substituting, adding, and multiplying

polynomials, solving equations, and simplifying shapes. Generalization involves the process of discovering patterns or shapes. Algebra is constructed by identifying patterns in a group of objects. Finally, the global meta-level involves solving the problems, modeling, and the proof of the theorem, which is unnecessary for the algebra symbols. Six forms of mathematical reasoning contribute to the student's success in algebra: generalization, abstraction, analysis, dynamics, modeling, and organization [2]. Generalization, functional relation, numbers properties and operations, and treatments of algebra numbers are the algebraic thinking components taught directly and systematically in the classroom [4]. Using symbols and algebra connections, various representations, patterns, and generalizations shape three components of algebraic thinking [5]. In addition, algebraic thinking has three components: modeling, general arithmetic, and function. Modeling includes open sentences, equality, using variables, and the meaning of equal sign. General arithmetic includes efficient numeric manipulation, simplifying calculation using connection figures, relational thinking strategies, and generalizations [6]. Algebraic thinking involves analysis of context through three activities: revealing information from a problem, presenting information mathematics in notation mathematics in text, charts, graphs, tables, and equations, and interpreting and applying findings mathematics [7].

Algebraic thinking is critical thinking that significantly helps students to solve the math problem. Algebra is essential to study because it is beneficial in daily life [3]. Besides, algebraic thinking also helps students to develop linguistics and representation to symbolize patterns, analyze, draw, and relationships [8]. Algebraic thinking is a process for the students to make comprehensive statements about symbols, patterns, and numbers in tables, pictures, diagrams, or mathematical expressions [9]. A better understanding of math is also needed for reasoning algebra [10]. In algebraic thinking, students are introduced to variables and symbols to simplify mathematical sentences into mathematical modeling [11]. In this study, researchers use algebraic thinking components developed by Blanton and Kaput: generalization of arithmetic, functional thinking, and generalization and justification.

There is a difference between receive and process for solving problems with thinking processes and algebra. Cognitive activities of the information obtained to produce new representations are part of thinking creativity [12]. Cognitive style refers to information about processing and receiving information [13]. Cognitive style is also a method of processing, receiving, and responding the problems. Besides, cognitive style is thinking about patterns and behavior that can influence learning and solution techniques [14]. Cognitive style systematically or intuitively significantly affects thinking, learning, and determining the solution strategies and defines by the liability to evaluate and solve problems [15].

Research that examines the ability of PMTs' algebraic thinking associated with systematic-intuitive cognitive style is still limited. Therefore, this research focuses on revealing the PMTs' algebraic thinking ability in terms of systematic-intuitive cognitive style.

METHOD

This study used the qualitative approach with the case study method. Thirty-one PMTs at one of the private universities in Surakarta District, Central Java, Indonesia, participated as subjects. Three instruments were used to collect the data: the algebraic thinking test, the CSI questionnaire, and the interview protocol. The researchers adopt seven problems from the 2012

PISA Released Mathematics Items [16]. The researchers composed seven problems: one generalization arithmetic problem, three functional thinking problems, and three generalization and justification problems. In this study, researchers used algebraic thinking components developed by [4]. Before being used, two experts in mathematics education validated the algebraic thinking test instrument. Based on the validation, researchers revised the problems based on the validators' comments. Next, the researchers piloted the test instruments to 12 PMTs not included in the research subjects. After the revision, the test instrument is ready for collecting the data.

Furthermore, researchers used a systematic-intuitive questionnaire adopted the CSI questionnaire developed by [17] to classify the style of cognitive students. Then, the researchers used the interview protocol to reveal the PMTs' algebraic thinking processes deeply on each cognitive style. Experts have also validated the interview protocol before being used.

In this study, out of the seven problems, the researchers analyzed three problems that represent the algebraic thinking component according to Blanton and Kaput [4], as presented in Table 1. Question 1 (Q1) represents the generalization arithmetic problem to reveal the PMTs' abilities in determining the number of cars made from available materials. Question 2 (Q2) represents the functional thinking problem to reveal the PMTs' abilities to determine point corners from available patterns along the formula. Then, question 3 (Q3) represents the generalization and justification problem to reveal the PMTs' abilities to determine the amount of white and blue tiles and their formulation.

Table 1. Algebraic Thinking Problems

NO

QUESTION

Q1

A photograph of a traditional Indonesian toy car made from orange peel. The car's body is a curved, hollowed-out section of an orange peel. It has two small, round green fruits (likely lime or lime leaves) attached to the sides to serve as wheels. A thin wooden stick is inserted into the center of the peel to act as an axle or handle.

Toys car from orange peel is one of the traditional toys for Indonesian children. The toy maker planned to make the toy car from orange peel to share with the children in his neighborhood. The material to make these cars are detailed in the table below:

Material	Stick	Orange peel for the body car	Orange peel for tire
Materials needed to make a toy car	3	2	4
Materials available	27	19	30

Determine the number of toys car that can be made from the available materials!

Q2 Given the geometry pattern as follows.

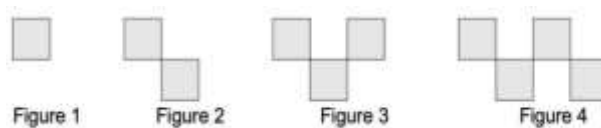
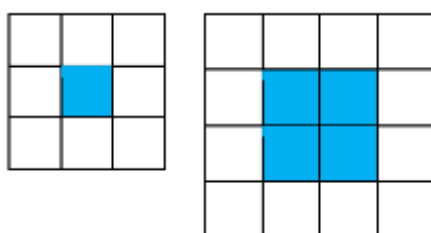


Figure 1 has 4 vertices, and Figure 2 has 7 vertices.

- If 30 squares are arranged according to the Figures 1 to 4 pattern, determine the number of vertices! Explain your method to solve the problem!
- Determine the formula to explain the relationship between the number of vertices and squares following the pattern in Figures 1 to 4!

Q3 A construction worker make a pattern of floor tile colored white and blue with the rule as in the Figure below.



Determine:

- The number of blue tiles to construct the floor tile pattern as in Figure if 52 white tiles are available!
- The number of white tiles to construct the floor tile pattern as in Figure if 64 blue tiles are available!
- The formulation of the relation between the number of blue and white floor tiles!

Based on the results of the algebraic thinking test and CSI questionnaire of 31 PMTs, the classification of PMTs' cognitive style is presented in Table 2.

Table 2. Summary of PMTs' Cognitive Style

Cognitive Style	Many Students
Systematic	3
Intuitive	6
Integration	7
Indifferent	1
Splits	14

Based on the data in Table 2, researchers selected four subjects: two systematic subjects and two subjects with intuitive cognitive styles. The criteria for subject selection is the result of an algebraic thinking test. In this study, all subjects obtained the highest scores on their test. To facilitate the data analysis, S1 and S2 coded the intuitive subjects, and systematic subjects were coded by S3 and S4.

Data analysis was performed first by document analysis, i.e., analysis of the PMTs' answer sheet in solving the problem related to algebraic thinking. In this step, researchers focused the

analysis on the PMTs' steps and strategies to solve the problems. The researchers used the assessment rubric in the analysis document in Table 3.

Table 3. Assessment Rubric Algebraic Thinking Instrument Test

Score	Assessment Criteria
0	The solution steps and the answers are incorrect
1	The partial solution steps are correct, but the answer is incorrect.
2	The solution steps are correct, but the answer is incorrect.
3	The solution steps and the answers are correct.

Furthermore, the researchers interviewed subjects about the solution steps and strategies for solving algebraic thinking problems. The interview is also intended to triangulate the PMTs' answers in documents, i.e., PMTs' answer sheets.

RESULTS AND DISCUSSION

Results

Q1 reveals the PMTs' abilities in solving the generalization arithmetic problem, which explores operations and relations on numbers and the relation between quantities. The results showed that S1, S3, and S4 solved the Q1 correctly. Meanwhile, S2 was unable to perform the solution steps properly. The example of the Q1 solution of S1 is presented in Figure 1.

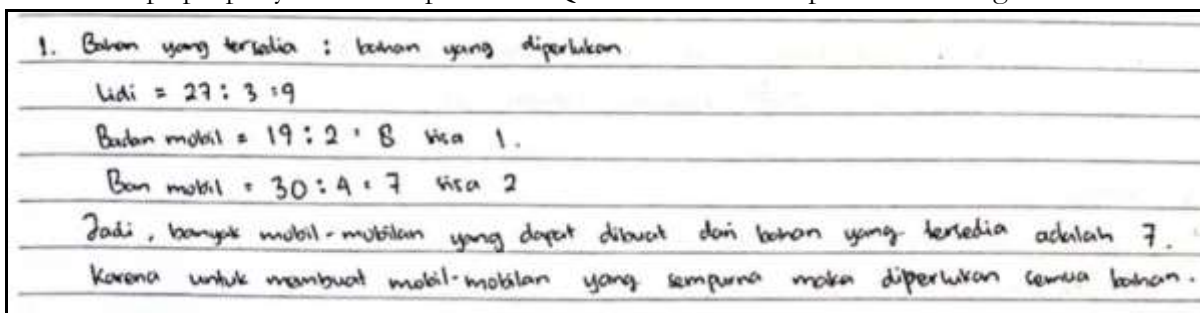


Figure 1. S1 Answer of Q1

Based on Figure 1, S1 used the strategy to determine the number of cars that can be made by dividing the available materials by the material needed to make a car. Based on this strategy, S1 divided the available stick, 27 by 3, to get 9. The number of available car bodies is 19 divided by 2 to get 8 remainders 1. Furthermore, the number of available car tires is 30 divided by 4 to get 7 remainders 2. Based on the calculation, S1 concluded that the number of cars that can be made with the available materials is seven.

Furthermore, the researchers interviewed S1 to reveal further the understanding to solve Q1. The excerpt of the interview with S1 is presented as follows (R: Researcher).

R: "For question number 1, please explain your solution steps?"

S1: " For the number 1, it is known the information about the number of sticks, car body, and car tires to make a toy car. I divide the stick by 3, the car body by 2, and the car tires by 4. After this calculation, seven toy cars can be made from all available materials."

P: " Why do you use the division method like that ?"

S 1: "The fast way, sir".

Based on the interview and document analysis results, it can be concluded that S1 can explore the operations and relations arithmetic for completing the problem-related generalization. Even though S1 demonstrates the ability to solve problem-related generalizations, however, S2 is unable to perform the solution steps properly. In other words, the intuitive subject's ability to solve the problem related to generalization varies.

Furthermore, the S4 answer in solving Q1 is presented in Figure 2.

① Bahan	Uk	Kulit untuk badan mobil	Kulit berbentuk ban
Yang diperlukan	3	2	4
Yang tersedia	27	19	30

Banyak mobil-mobilan yang dapat dibuat dari bahan yang tersedia?

Penglesaian:

- Bahan kulit untuk badan mobil yang tersedia dibagi dengan yang diperlukan
 $19 : 2 = 9 \text{ sisa } 1$
- Bahan kulit berbentuk ban
 $30 : 4 = 7 \text{ sisa } 2$
- Bahan lidi
 $27 : 3 = 9$

Jadi, banyak mobil yang dapat dibuat yaitu 7 mobil karena bahan kulit berbentuk hanya mencukupi untuk membuat 7 mobil. Atau apabila kulit badan mobil bisa dibuat berbentuk ban maka mobil yang dapat dibuat yaitu 9 dengan membuat 1 kulit untuk badan mobil menjadi 6 kulit berbentuk ban. Karena untuk membuat 9 mobil dibutuhkan 36 (9x4) kulit berbentuk ban, sehingga dari 30 bahan yang tersedia kurang 6 kulit berbentuk ban.

Figure 2. S4 Answers of Q1

Based on Figure 2, the S4 writes the method to determine the number of toy cars that can be made from available materials by dividing the available materials by the material needed for every toy car. S4 divides the available car body, which is 19 by 2 to get 9 remaining 1, the car tires, 30 by 4 to get 7 remaining 2, and the stick, 27 by 3 to get 9. Based on the calculation, S4 concludes that the number of toy cars that can be made based on available materials is seven.

Afterward, the researcher interviewed to reveal S4's understanding of the solution steps in solving Q1. The excerpt of the interview is presented as follows.

R: " Explain your strategy to solve Q1?"

S4: " For the number 1, the material needed to make a car is known. So, based on the available material, I try to calculate the possibility of the number of toy cars that can be made. I divide the stick by 3, the car body by 2, and the car tires by 4. After this calculation, seven toy cars can be made from all available materials."

Based on the interview and document analysis results, it can be concluded that S4 can explore operations and relations arithmetic to solve a problem-related generalization. In other words, the systematic subject can solve the problem related to generalization.

Subjects in solving Q1 use two strategies. The researchers called the first strategy used by the intuitive subject is direct division method. In this method, the intuitive subject directly divided the available materials by the materials needed to determine the toy cars that can be made. After this, researchers called the second strategy used by systematic subjects the systematic

method. In this strategy, systematic subjects solve Q1 by using problem-solving steps. First, the subjects present the information on Q1 in the form of a table. Then, subjects write what is asked in Q1. Afterward, subjects solve the problem by dividing the available materials and materials needed to make a toy car. Finally, subjects conclude from the calculation results.

The problem of number 2, Q2, is used to analyze the PMTs' ability to functional thinking, which is a process for finding geometry patterns. Based on the test results, all subjects can solve Q2 using the appropriate solution steps and obtain the correct answer. The S2 answer in solving Q2 is presented in Figure 3.

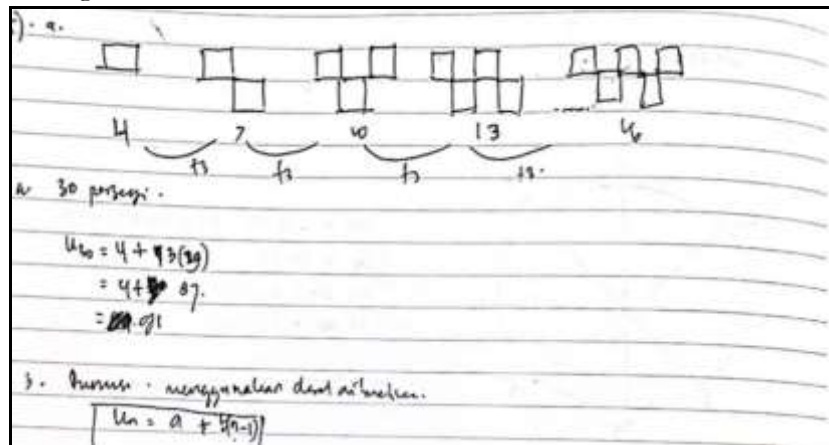


Figure 3. S2 Answers of Q2

Based on Figure 3, S2 formerly drew the square pattern to determine the number of vertices on the pattern formed. Then, S2 determines the difference in the number of vertices between patterns is 3. Hence, S2 determines the number of vertices if a pattern is formed by n squares using the arithmetic sequences: $U_n = a + (n - 1)b$, while a = the initial vertices and b = the difference in the number of vertices between patterns. For $n=30$, by substituting $a = 4$, $(n - 1) = 29$, and $b = 3$, the result is 91. Finally, S2 determines the number of vertices if n squares form a pattern is $U_n = a + (n - 1)b$.

Furthermore, researchers interviewed to reveal S2's understanding of the solution steps in solving Q2. The excerpt of the interview is presented as follows.

R: "Explain your strategy to solve Q2?"

S2: "Number 2 (silent to read the question). It is already known that the number pattern in the question. For example, there are 30 squares arranged following the square pattern formed. The first square pattern includes 4 vertices. The next patterns include 7, 10, 13, and 16 vertices, so the difference in vertices between patterns is 3. So, for 30 squares pattern can be determined by $U_{30} = 4 + 3(30 - 1)$, so $U_{30} = 4 + (3 \times 29)$, and $4 + 87 = 91$. The formula is $U_n = a + (n-1)b$."

Based on the interview and document analysis, it can be concluded that S2 can demonstrate functional thinking ability, a process for finding geometry patterns.

Subsequently, the S3 answer to the Q2 problem is presented in Figure 4.

Pola titik sudut :

4 7 10 13 ...

 +3 +3 +3

(a) 30 persegi.

Dapat dihitung dengan rumus barisan aritmetika.

Dengan $a = 4$ dan $b = 3$. Dik. $n = 30$.

Banyak titik sudut = $a + (n-1)b$

$$= 4 + 29 \cdot 3$$

$$= 4 + 87$$

$$= 91 //$$

(b) Rumus :

Banyak titik sudut = $a + (n-1)b$.

Dengan $a = \text{titik sudut pada persegi pertama}$.

$b = \text{selisih jumlah titik sudut persegi pertama dan kedua}$.

$n = \text{banyak persegi}$.

Figure 4. S3 Answers of Q2

Based on Figure 4, S3 determines firstly the number of patterns that represent the number of vertices and the differences of vertices on each pattern. To determine the number of vertices for the 30 squares pattern, S3 used variables a to represent the number of vertices on the first pattern, b to represent the differences between patterns, and n to represent the number of squares on the pattern. Then, S3 formulates the pattern formed as the arithmetic sequences, $a + (n - 1) b$. Afterward, S3 substitutes $a = 4$, $b = 3$, and $n = 30$ into the formula to obtain 91. Finally, S3 can correctly formulate the number of vertices for the n squares pattern.

Furthermore, the researchers interviewed to reveal the S3 understanding of the solution steps in solving Q2. The excerpt of the interview is presented as follows.

R: "How is your strategy to solve Q2?"

S3: "I found patterns 4, 7, 10, 13, so I used the arithmetic sequence with $a = 4$, $b = 3$. If there are 30 squares, that is $n = 30$. We can directly substitute to the arithmetic sequence formula."

Based on the interview and document analysis, it can be concluded that S3 has the ability of functional thinking, a process to solve problems related to geometry patterns.

There are two strategies carried out by subjects to solve the Q2 problem. The intuitive subject, S2, uses the picture to determine the pattern. S2 first visualizes the square pattern by drawing the first, second, third, fourth, and fifth patterns. Subsequently, S2 identified the number of vertices on each pattern and the differences of vertices between patterns to formulate the pattern for n squares, that is, the arithmetic sequences $U_n = a + (n-1) b$. In contrast, the systematic subject, S3, uses the number pattern to identify the n squares pattern.

Question number 3, Q3, is used to reveal the PMTs' ability in generalization and justification, which is a process for finding a pattern or shape. Based on the test results, S2 and S4 can solve Q3 using appropriate solution steps and obtain the correct answer. Meanwhile, S1 and S3 partly solve the Q3 using appropriate solution steps so that the answers are incorrect. Figure 5 shows the S2 answer in solving Q3.

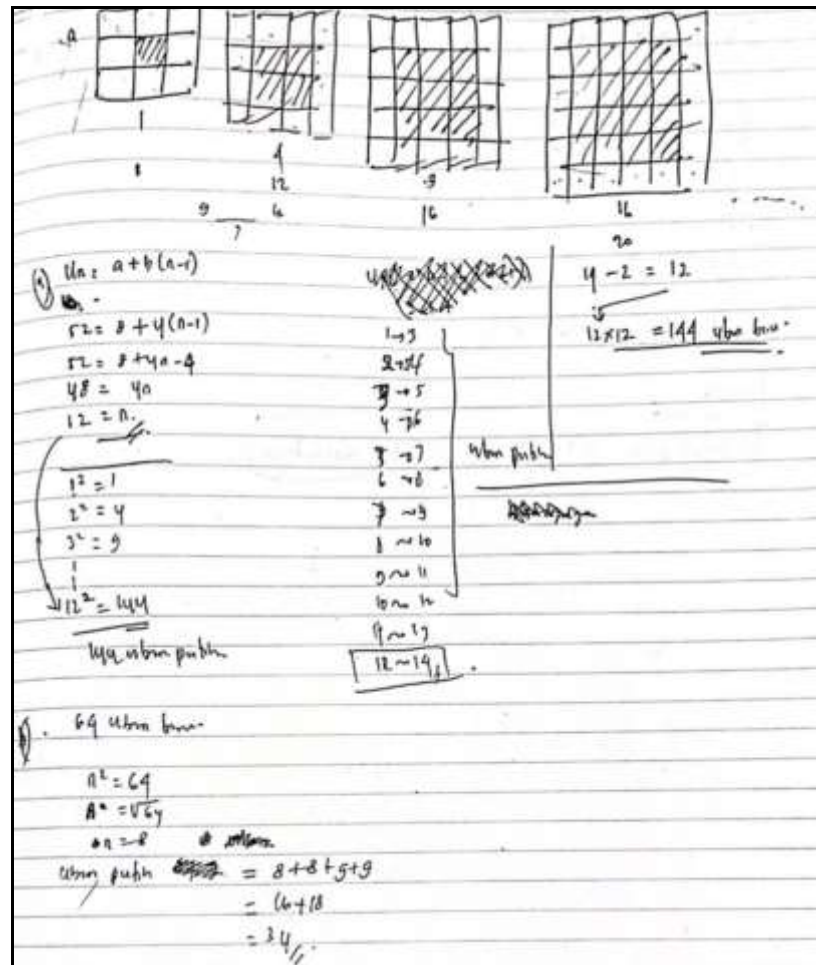


Figure 5. S2 Answers of Q3

Based on Figure 5, S2 used the picture to determine the pattern of the white and blue tiles. Firstly, S2 drew the first, second, third, and fourth patterns to visualize the tiles' pattern. Secondly, S2 identified the number of blue and white tiles on each pattern, that is 1 blue tile and 8 white tiles for the first pattern, 4 blue tiles and 12 white tiles for the second pattern, 9 blue tiles and 16 white tiles for the third pattern, and 16 blue tiles and 20 white tiles for the fourth pattern. Then, S2 determines the number of blue tiles using an arithmetic sequence if 52 white tiles are available. Based on the calculation, the result is $n = 12$. S2 can also identify that the number of blue tiles for the first pattern is $1^2 = 1$, the second is $2^2 = 4$, and the third is $3^2 = 9$. So, the number of blue tiles is n^2 for the n th pattern. Then, for $n = 12$, the number of blue tiles is $12 \times 12 = 144$. For the 3b question, S2 can determine n if 64 blue tiles are available by determining the square root of 64, 8. Unfortunately, S2 made an error in determining the number of white tiles for the n^{th} pattern by calculating $8 + 8 + 9 + 9$ to obtain 34.

Furthermore, the researchers interviewed to reveal the S2 understanding of the solution steps in solving Q3. The excerpt of the interview is presented as follows.

Q: "Explain your strategy to solve Q3."

S2: "In question number 3, ask the number of tiles on the pattern. There is one blue tile for the first pattern and 4, 9, and 16 blue tiles for the next pattern. So, the solution can be solved by the formula $U_n = a + (n - 1) b$. (silent) $U_n = 52$, which means that there are 52

white tiles. Hence, $a = 8$ is the white tiles in the first pattern. Then $52 = 8 +$ the differences in the number of white tiles between patterns that is 4. After calculation of U_n , obtain $n = 12$. Then, the blue tiles can be determined by squaring 12 to obtain 144. For the solution of number 3b, the number of white tiles required if there are 64 blue tiles, for instance, n^2 represent the number of blue tiles, so for 64 blue tiles, the value of n is 8. Hence, the number of white blue can be calculated by adding the number of white tiles on each side of the square, that is $8 + 8 + 9 + 9$, so $16 + 18 = 34$ "

Q: "Why can it be $8 + 8$, then $9 + 9$ like that?"

S2: "That is 8 from the right side of the square of tiles, and then 8 from the left side, but the above side should be 10 and the bottom side also 10 white tiles. That is the error for 9 on the above and bottom sides. Hence, the number of white tiles is 36."

Based on the interview and document analysis, it can be concluded that S3 demonstrates the ability to use patterns in solving problem-related for generalization and justification.

Furthermore, the S4 answer in solving Q3 is presented in Figure 6.

The figure shows a handwritten mathematical solution for Q3. It begins with a table summarizing tile counts for three patterns:

Banyak ubin biru	1	4	9
Banyak ubin putih	8	12	16
Total ubin	9	16	25

Below the table, the solution is divided into three parts:

a) Jika tersedia 52 ubin putih, ubin biru berapa?

- Banyak ubin biru berpola yaitu 1, 4, ...
- Rumusannya yaitu disisalkan $b = n^2$
- Banyak ubin putih berpola yaitu 8, 12, ...
- Rumusannya yaitu disisalkan $p = 4(n+1)$
- Total ubin juga berpola yaitu 9, 16, ...
- Rumusannya yaitu disisalkan $x = (n+2)^2$

Then, it solves for n using the given 52 white tiles:

$$p = 4(n+1)$$

$$52 = 4n + 4$$

$$48 = 4n \rightarrow n = 12$$

Finally, it calculates the number of blue tiles:

$$b = n^2$$

$$= 12^2$$

$$= 144 \text{ ubin biru}$$

b) Banyak ubin putih, jika tersedia 64 ubin biru

It sets up the equations:

$$b = n^2$$

$$64 = n^2$$

$$8 = n$$

Then it calculates the number of white tiles:

$$p = 4(n+1)$$

$$= 4(8+1)$$

$$= 4 \cdot 9$$

$$= 36 \text{ ubin putih}$$

c) Rumusnya

- Banyak ubin biru (b) = n^2
- Banyak ubin putih (p) = $4(n+1)$
- Total ubin (x) = $b + p$

It then shows the derivation of the total formula:

$$= n^2 + 4(n+1)$$

$$= n^2 + 4n + 4$$

$$= (n+2)^2$$

Figure 6. S4 Answers of Q3

Figure 6 shows that S4 first represents the picture pattern of the problem in verbal representation in the tabular form. Furthermore, based on the number pattern table compiled, S4 determines the formula for each pattern obtained, namely the number of blue tiles with n^2 , the number of white tiles $4(n-1)$, and the total number of tiles with $(n+2)^2$. Based on the formula obtained, S4 can determine the number of blue tiles if there are 52 white tiles available using the formula $p=4(n-1)$ and substitution $p=52$ to get $n=12$. Next, S4 uses the formula $b = n^2$ to determine the blue tile. By substitution $n=12$, the number of blue tiles is 144. Then, in solving problem number 3b, that is, the number of white tiles needed if there are 64 blue tiles available, S4 uses the formula $b = n^2$. For $b=64$, $n = 8$ is obtained. Then, S4 uses the formula $p=4(n-1)$ to determine the number of white tiles. Substitution $n = 8$ obtained $p = 4(8 + 1)=36$. Thus, if there are 64 blue tiles, the number of white tiles is 36.

Furthermore, the researchers conducted interviews with S4 to reveal further S4's understanding of the steps to solving the Q3 question. Excerpts from the interview with S4 are presented as follows.

Q : "Explain how to solve problem number 3?"

S4 : "I determine first the number of blue tiles, the number of white tiles, and the total number of tiles from Figure 1 and Figure 2. Then, I also try to find the pattern in Figure 3. It forms a pattern of blue, white, and total tiles. In the solution of number 3a, if there are 52 white tiles available, it asks how many blue tiles. I suppose that the white tiles are $p = 4(n + 1)$, then the pattern of total tiles is 9, 16, 25, and so on. I also assume the formula with $x = (n + 2)^2$. If there are 52 white tiles, how many blue tiles are there? Look for the 52 white tiles; which picture pattern is this? then substituted into the formula $p = 4(n + 1)$ we get $n = 12$. Therefore, the number of blue tiles in Figure 12 uses the formula $b = n^2$, meaning that 12^2 yields 144 blue tiles. Solving problem 3b is also similar to solving problem 3a. In problem 3b, there are 64 blue tiles. First, substituted into the formula $b = n^2$, namely $64 = n^2$, obtained $n = 8$, it means that the 8th picture pattern. Then the number of white tiles in the 8th picture pattern is $p = 4(n + 1)$, equal to $4(8 + 1)$ so that 36 white tiles are obtained."

Based on interviews and document analysis, it can be concluded that S4 can use patterns to solve problems related to generalization and justification. In this case, S4 presents the picture pattern of the problem in the form of a verbal representation to solve the problem.

The subject uses two strategies to solve question 3 regarding generalization and justification. The first strategy is carried out by intuitive subjects, namely, using visual representation to identify patterns. The second strategy is carried out by systematic subjects, namely by presenting picture patterns in the form of verbal representations and patterns in tables and sentences.

Discussion

The results of the data analysis showed that the subject's ability to solve problems related to the components of algebraic thinking with intuitive and systematic cognitive styles varied. The two intuitive subjects have different abilities in solving problems related to arithmetic generalizations as well as generalizations and justifications. However, on questions related to functional thinking, the two intuitive subjects demonstrated the ability to solve them correctly.

Furthermore, both systematic subjects demonstrated the ability to solve problems related to arithmetic generalizations and functional thinking. However, in solving questions related to generalization and justification, the abilities of the two subjects differed.

In addition to differences in the ability to solve problems for each component of algebraic thinking, the strategies used by intuitive and systematic subjects are also different. On problems related to arithmetic generalizations, intuitive subjects immediately perform calculation operations from the data in the questions. Meanwhile, systematic subjects are more systematic in solving problems using problem-solving procedures from Polya. Furthermore, intuitive subjects tend to solve problems related to functional thinking components as well as generalizations and justifications using picture patterns. Meanwhile, systematic subjects tend to use number patterns to solve problems.

Research related to differences in abilities and strategies in solving problems in intuitive and systematic subjects is also supported by previous research. Research by [18] showed differences in algebraic thinking activities in subjects with systematic and intuitive cognitive styles in identifying and describing pattern components. The systematic subject determines the differences between terms in the sequence in identifying number patterns, while the intuitive subject did not use this step. Furthermore, systematic subjects use more systematic steps when determining each term in a number sequence, while intuitive subjects use trial and error. Research by [19] also showed that there are different approaches to relational problem-solving used by students with a systematic and intuitive cognitive style. Students with systematic and intuitive cognitive styles can fulfill all indicators of problem-solving. However, intuitive subjects solve problems in ways that cannot be predicted by using more straightforward calculations. Furthermore, the results of other studies also showed that subjects with a systematic cognitive style have more systematic and detailed characteristics by paying attention to every piece of information in the problem as a whole in solving math problems [20]. In contrast, intuitive subjects use a shorter way of solving problems.

Other research by [21] also showed that students with a systematic cognitive style like order and detail take longer to solve problems. In contrast, students with intuitive cognitive styles tend to solve math problems quickly and without much thought. Students with a systematic cognitive style recognize all information and record it on the answer sheet, while students with an intuitive cognitive style recognize all information but do not record it [22]. Another difference is found in students' systematic subjects solving questions using tables, while intuitive subjects solve problems with picture representations. In line with research by [23], subjects with a systematic cognitive style in solving mathematical problems tend to represent the problems verbally, while subjects with an intuitive cognitive style tend to represent the problems visually.

The development of algebraic thinking is needed to help students solve math problems, especially those related to problems in everyday life. Therefore, teachers must use appropriate learning strategies to develop students' algebraic thinking skills. Learning that emphasizes the development of ideas and encourages students to be more active in problem-solving can help improve students' algebraic thinking skills [24].

CONCLUSION

PMTs' algebraic thinking abilities regarding systematic and intuitive cognitive styles are diverse. On problems related to arithmetic generalizations, systematic subjects demonstrated the ability to solve problems correctly and use systematic solution steps, whereas intuitive subjects have more diverse abilities and use solving strategies using direct division. On problems related to functional thinking, both systematic and intuitive subjects demonstrated the ability to solve problems. However, there are differences regarding solution strategies. Intuitive subjects use pictures or visual representations to identify patterns, while systematic subjects use number patterns or numerical or verbal representations. Then, on problems related to generalization and justification, the ability of systematic and intuitive subjects varies. However, the solving strategy used is almost the same as in functional thinking problems, namely, using visual representations or picture patterns on intuitive subjects and numerical or verbal representations on systematic subjects. Although the results of this study provide valuable information regarding PMTs' algebraic thinking skills based on cognitive style, further identification regarding the factors that influence the diversity of abilities and problem-solving strategies related to algebraic thinking needs to be studied further.

REFERENCES

- [1] C. Kieran, "Algebraic Thinking in the Early Grades : What Is It?," *Math. Educ.*, vol. 8, no. 1, pp. 139–151, 2004.
- [2] H.-C. Lew, "Developing Algebraic Thinking in Early Grades: Case Study of Korean Elementary School Mathematics," *Math. Educ.*, vol. 8, no. 1, pp. 88–106, 2004.
- [3] G. Booker and W. Windsor, "Developing algebraic thinking: Using problem-solving to build from number and geometry in the primary school to the ideas that underpin algebra in high school and beyond," *Procedia - Soc. Behav. Sci.*, vol. 8, no. 5, pp. 411–419, 2010, doi: 10.1016/j.sbspro.2010.12.057.
- [4] J. J. Kaput and M. Blanton, "Characterizing a classroom practice that promotes algebraic reasoning," *J. Res. Math. Educ.*, vol. 36, no. 5, p. 412, 2005.
- [5] J. Dindyal, "Algebraic thinking in geometry at high school level: students' use of variables and unknowns," *Natl. Inst. Educ.*, vol. 27, no. 21, pp. 183–190, 2004, <http://www.ncbi.nlm.nih.gov/pubmed/4428755>.
- [6] N. C. Ralston, "The Development and Validation of a Diagnostic Assessment of Algebraic Thinking Skills for Students in the Elementary Grades," vol. 16, no. 22, pp. 119–128, 2013.
- [7] R. Chairunisa, M. Maimunah, and Y. Roza, "Proses Berpikir Aljabar Siswa SMP Ditinjau Dari Kemampuan Akademik Matematika Dan Gender," *J. Inspirasi Pendidik.*, vol. 10, no. 2, pp. 85–95, 2020, doi: 10.21067/jip.v10i2.4347.
- [8] M. L. Blanton and J. J. Kaput, "Functional Thinking as a Route Into Algebra in the Elementary Grades," 2011, pp. 5–23.
- [9] U. Silma, "Analisis Kemampuan Berpikir Aljabar Siswa dalam Model Pembelajaran Learning Cycle 5E," *Elektron. Pembelajaran Mat.*, vol. 5, no. 3, pp. 300–318, 2018.
- [10] S. Kriegler and S. Oaks, "Submitted for Algebraic Concepts in the Middle School By 3502 Weslin Avenue About the author : Dr . Shelley Kriegler teaches mathematics education

courses at UCLA and is an educational consultant for Creative Publications . Just WHAT IS ALGEBRAIC THINKING,” 1997.

- [11] I. Istikomah, E. P. Astuti, and H. Kurniawan, “Kemampuan Berpikir Aljabar Siswa Climber dalam Menyelesaikan Masalah SPLDV,” *AlphaMath J. Math. Educ.*, vol. 6, no. 2, p. 96, 2020, doi: 10.30595/alphamath.v6i2.8117.
- [12] A. A. Nugroho, N. Nizaruddin, I. Dwijayanti, and A. Trisianti, “Exploring students’ creative thinking in the use of representations in solving mathematical problems based on cognitive style,” *JRAMathEdu (Journal Res. Adv. Math. Educ.)*, vol. 5, no. 2, pp. 202–217, 2020, doi: 10.23917/jramathedu.v5i2.9983.
- [13] U. Umah, “Comparison of Students’ Covariational Reasoning Based on Differences in Field-Dependent and Field-Independent Cognitive Style,” *Numer. J. Mat. dan Pendidik. Mat.*, vol. 4, pp. 41–54, 2020, doi: 10.25217/numerical.v4i1.638.
- [14] S. R. I. Ramakrishna and M. Vidyalaya, “Educational Extension,” vol. 52, no. 3, 2015.
- [15] P. G. W. Keen, *Cognitive Style & The Problem-Solving Process: An Experiment*. 1974.
- [16] OECD, “PISA 2012 Released Mathematics Items,” *OECD Publ.*, pp. 1–92, 2013, [Online]. Available: <https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf>.
- [17] L. P. Martin, *The Cognitive-Style Inventory*, vol. 8, no. 1. 1998.
- [18] F. F. Arina and A. Lukito, “Algebraic Thinking of Junior High School Students in Solving Number,” *MATHEdunesa J. Ilm. Pendidik. Mat.*, vol. 8, no. 1, pp. 71–77, 2019.
- [19] I. N. Khoyimah, “Profil Berpikir Relasional Siswa Smp Dalam Menyelesaikan Masalah Matematika Ditinjau Dari Gaya Kognitif Sistematis-Intuitif,” *MATHEdunesa*, vol. 10, no. 2, pp. 396–409, 2021, doi: 10.26740/mathedunesa.v10n2.p396-409.
- [20] C. Sa, S. Irawati, and E. Hidayanto, “Systemic Thinking Processes Of Preservice Teachers With Systematic Cognitive Style In Solving Complex Problems,” *J. Posit. Sch. Psychol.*, vol. 2022, no. 5, pp. 2706–2730, 2022, [Online]. Available: <http://journalppw.com>.
- [21] M. Widayanti, Y. Jumiah, and R. Ijuddin, “Penalaran Proporsional Siswa SMP Negeri 18 Pontianak,” *J. Pendidik. dan Pembelajaran Khatulistiwa*, vol. 9, no. 10, pp. 1–9, 2020.
- [22] D. M. N. Fadilla and T. Y. E. Siswono, “Penalaran Proporsional Siswa Bergaya Kognitif Sistematis dan Intuitif Dalam Menyelesaikan Masalah Numerasi,” *MATHEdunesa*, vol. 11, no. 3, pp. 630–643, 2022, doi: 10.26740/mathedunesa.v11n3.p630-643.
- [23] H. Y. W. Kristanto and J. T. Manoy, “Representasi Matematis Siswa SMA dalam Menyelesaikan Masalah Matematika Ditinjau dari Gaya Kognitif Sistematis dan Intuitif,” *J. Penelit. Pendidik. Mat. Dan Sains*, vol. 4, no. 2, p. 50, 2021, doi: 10.26740/jppms.v4n2.p50-59.
- [24] M. Riskon, R. Rochmad, and N. R. Dewi, “Unnes Journal of Mathematics Education Research Algebraic Thinking Ability with Creative Problem Solving Integrated 4C Model Viewed from Mathematical Disposition,” *Unnes J. Math. Educ. Res.*, vol. 10, no. 87, pp. 38–53, 2021, [Online]. Available: <http://journal.unnes.ac.id/sju/index.php/ujmer>.