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Performance Quantile Regression and Bayesian Quantile Regression in Dealing with Non-normal Errors (Case Study on Simulated Data)

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Abstract

This research discusses the performance of quantile regression and Bayesian quantile regression methods. Quantile regression uses parameter estimation by maximizing the value of the likelihood function, while Bayesian quantile regression uses parameter estimation with the Bayesian concept. The Bayesian concept in question looks for solutions from the posterior distribution with Gibbs Sampling. The purpose of the study is to compare the two methods. The data used is simulated data with a total of 100 generated data. The results obtained by the Bayesian quantile regression method are superior to the indicator used MSE with the result of 1.7445. The smallest MSE value is obtained in the model that is in quantile of 0.5.

INTRODUCTION

Regression analysis is a statistical method used to determine the relationship between one independent variable (independent) and one or more dependent variables (dependent) [1],[2], [3],[4]. Through this approach, researchers can predict the value of the dependent variable based on the value of the independent variable, as well as evaluate how strong and significant the relationship between them is. Regression analysis is often used in various fields such as economics, business, and social sciences for data-driven decision-making. By utilizing regression models, patterns and trends in data can be identified more systematically and objectively.

The classical assumptions that must be met in regression analysis are important conditions that ensure the validity of the model estimation results. These assumptions include residual normality, homoscedasticity (constant residual variance), absence of autocorrelation, absence of multicollinearity, and linear relationship between independent and dependent variables. If these assumptions are met, the regression parameter estimates will be BLUE (Best Linear Unbiased Estimator), which means they have high efficiency and accuracy in predicting the value of the dependent variable. Violation of classical assumptions can lead to misleading conclusions in data-based decision making [5],[6].

Quantile regression is a statistical analysis method used to understand the relationship between the independent variable and the dependent variable at various quantile points of the data distribution, rather than just at the mean like ordinary linear regression [6],[7]. This technique is particularly useful when data has outliers, non-normal distributions, or when the relationship between variables is different at different quantile levels. By using quantile regression, researchers can get a more complete picture of the effect of the independent variable on the entire distribution of the dependent variable. This allows for more flexible analysis and is robust to various forms of data irregularities.

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The main advantage of quantile regression compared to classical regression is its ability to provide a more complete picture of the relationship between independent and dependent variables, not only at the mean, but also at various points in data distribution. Classical regression, like ordinary linear regression, only models the mean of the dependent variable, making it less effective when the data contains outliers or non-normal distribution [8]. In contrast, quantile regression is more robust to outliers and can reveal different dynamics at different quantiles, such as differences in the effect of variables on groups with low or high values. This makes quantile regression very useful in the analysis of heterogeneous or asymmetric data. One disadvantage of quantile regression is its sensitivity to multicollinearity among the independent variables, which may cause the estimation results to be unstable or difficult to interpret. In addition, quantile regression also requires a large sample size to produce reliable estimates, especially when estimating extreme quantiles (such as the 5th or 95th quantile). The calculation process is also more complex than ordinary linear regression, as it does not have a closed analytic solution and relies on numerical methods, which requires more computational time. The weaknesses of quantile regression can be overcome with Bayesian quantile regression.

Bayesian quantile regression is a statistical approach that combines quantile regression methods with Bayesian inference principles to model the relationship between independent variables and various quantiles of the dependent variable distribution [9]. In this method, parameter uncertainty is accounted for through the posterior distribution, allowing for more flexible and informative estimation than classical quantile regression. This approach is particularly useful when the residual distribution is not normal or when significant outliers affect the average regression result. By using priors and posteriors, Bayesian quantile regression provides a powerful probabilistic framework for the analysis of heterogeneous and asymmetric data [10].

Bayesian quantile regression combines traditional quantile regression principles with a Bayesian framework using Asymmetric Laplace Distribution (ALD) density functions as the base likelihood. The seminal approach by Yu and Moyeed [11], introduced the ALD model to estimate quantiles by assigning priors to the regression coefficients, thereby allowing complete probabilistic inference and credible intervals for the quantile parameters [12]. Since then, various developments have emerged, including hierarchical quantile regression for multilevel data, dynamic quantile regression models for time series, and Bayesian spatial models to handle space dependencies. MCMC methods, such as Gibbs sampling and Metropolis Hastings, are often used to extract posterior samples, while variational Bayes techniques are gaining popularity to speed up computation on large datasets. In addition, the integration of penalty priors, such as Laplace supports variable selection and regularization, making Bayesian quantile regression a flexible tool for robust analysis under various conditions of heteroscedasticity and outliers [13].

METODS

The research to achieve the stated objectives. The approach was designed to ensure the validity and reliability of the results, taking into account the context and characteristics of the research data.

1. Quantile Regression

Quantile Regression (QR) is a method that estimates a distribution of data across different quantiles [14]. Regression is Quantile Regression (QR) is a regression technique used to estimate the relationship between an independent variable and a quantile of the distribution of the dependent variable. Suppose $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)'$is a vector the dependent variable and $\boldsymbol{x} = (x_1, x_2, \ldots, x_k)'$ is the independent variable, then the τ^{th} is quantile, for $0 < \tau < 1$, if *n* samples and *k* predictors for $i = 1,2, \ldots, n$ is formula below [7]:

$$y_i = \beta_{0\tau} + \beta_{1\tau}x_{i1} + \beta_{2\tau}x_{i2} + \cdots + \beta_{k\tau}x_{ik} + \varepsilon_i \tag{1}$$

With $\boldsymbol{\beta}(\tau)$ as the parameter vector and $\boldsymbol{\varepsilon}$ as the residual vector. The conditional quantile function τ^{th} is $Q_\tau(\boldsymbol{y}|\boldsymbol{x}_i) = \boldsymbol{x}_i'\boldsymbol{\beta}_\tau$, to estimate $\widehat{\boldsymbol{\beta}_\tau}$ ekuivalent with minimize the equation below[15]:

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$$\sum_{i=1}^{n} \rho_\tau(y_i - \boldsymbol{x}_i'\boldsymbol{\beta}_\tau). \tag{2}$$

for $\rho_\tau(u) = u(\tau - I(u < 0))$is the *loss function* with the equation [16]:

$$\rho_\tau(\epsilon) = \varepsilon\big(\tau I(\varepsilon \geq 0) - (1-\tau)I(\varepsilon < 0)\big). \tag{2. 3}$$

$I\ (.)$ is an indicator function, which has a value of 1 when $I\ (.)$ is true dan 0 otherwise.

2. **Bayesian**

Bayes theorem is the foundation of the Bayesian method, developed by Thomas Bayes in the 18th century. This theorem describes how we can update the probability of a hypothesis or parameter based on newly obtained evidence. Bayes theorem is formulated by equation below:

$$P((A|B) = \frac{P((B|A)P(A)}{P(B)} \tag{3}$$

Where:
$P((A|B)$ is posterior distribution
$P((B|A)$ is likelihood function
$P(A)$ is prior distribution
$P(B)$ is probabilitas marginal (total probability from data (B) not depent to A.

Definition 1. [17] Joint probability density function of random variables $X_1, X_2, \dots, X_n$ calculated at $x_1, x_2, \dots, x_n$ is $f(x_1, x_2, \dots, x_n|\beta)$ which is the likelihood function. For a given value of $x_1, x_2, \dots, x_n$ the likelihood function is a function of the parameter β which can be denoted by $L(\beta)$. If $X_1, X_2, \dots, X_n$ are mutually independent random samples of $f(x;\beta)$ then:

$$L(\beta) = f(x_1;\beta)\, f(x_2;\beta)\ \dots f(x_n;\ \beta) \tag{4}$$
$$= \prod_{i=1}^{n} \boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{\beta})$$

The prior distribution is the initial distribution that provides information about the parameters to be estimated. In estimating a parameter value, the prior distribution can be chosen subjectively by the researcher. The prior distribution is divided into two, namely [18],[19]:

a. Related to the distribution form of the results of identifying the data pattern obtained from the likelihood function, namely:
   1. Conjugate prior distribution. This prior distribution is determined based on the choice of priority in a model considering the likelihood function.
   2. Non-conjugate prior distribution. Giving priors to the model ignores the pattern of forming the likelihood function.

b. Related to previous information related to the determination of each parameter in the prior distribution pattern, namely:

1. Informative prior distribution. This prior distribution refers to the assignment of parameters from the prior distribution that has been selected, both the conjugate prior distribution and the non-conjugate prior.
2. Non-informative prior distribution. This prior distribution is not based on existing data or a prior distribution that contains no information about the parameters.

Definition 2. The conditional probability density function of the parameter β given observations $x = x_1, x_2, \dots, x_n$, is expressed as the posterior probability density function given by [20]:

$$f(\beta|x) = \frac{L(\beta)f(\boldsymbol{\beta})}{\int_{-\infty}^{\infty} L(\beta)f(\beta)d\beta} \tag{5}$$

Since the function in the denominator tends to be constant because it does not depend on the value of β, the posterior probability density function can be written as follows [20]:

$$f(\beta|x) \propto L(\beta)f(\boldsymbol{\beta}) \tag{6}$$

Definition 3: [21] The mean of the posterior distribution $f(\beta|x)$ expressed by $\hat{\beta}$ is referred to as the Bayes estimator for β.

3. Bayesian Quantile Regression

The Bayesian concept above will be used as the basis for estimating parameters in the Bayesian concept. The Bayes method combines the likelihood function with the prior distribution of the parameters to obtain the posterior distribution which is the basis for estimating the parameters. Bayesian quantile regression method (BQR) is a combination of quantile regression method with Bayesian approach. The combination [22] suggested Bayesian quantile regression combines traditional quantile regression principles with a Bayesian framework through the use of Asymmetric Laplace Distribution (ALD) density functions as the base likelihood. The ALD distribution is one of the continuous probability distributions. A random variable ϵ with ALD distribution with likelihood density function $f(\epsilon)$, namely [11]:

$$f_\tau(\epsilon) = \tau(1-\tau)exp\,(-\rho_\tau(\epsilon)) \tag{7}$$

With $0 < \tau < 1$ and ρ_τ is a loss function with as the error of the estimation and is an indicator function. Given an observation $y = (y_1, y_2, \dots, y_n)$ to incorporate the quantile regression method into the Bayesian approach in estimating the parameters β ALD is used as the likelihood function which is expressed as follows:

$$L(\beta) = \tau^n(1-\tau)^n exp\{-\textstyle\sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta\} \tag{8}$$

The posterior distribution β of is obtained by multiplying the likelihood function in equation (8) with the posterior distribution so that the posterior β distribution is $p(\beta)$ obtained as follows:

$$f(y) \propto L(\beta)p(\beta)$$

$$= \tau^n(1-\tau)^n \exp\{-\textstyle\sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta\}p(\beta) \tag{9}$$

In determining the posterior distribution for the estimated parameters on the use of ALD as a *likelihood* function for the data directly is difficult to solve analytically [15]. To overcome this difficulty, a numerical approach is used with the help of the MCMC (Markov Chain Monte Carlo) algorithm which is not only effectively used but also able to overcome complex analytical integration [12][23]. MSE stands for Mean Squared Error, It is a common metric used to measure the average of the squares of the errors that is, the average squared difference between the predicted values and the actual values.below [14][20]:

$$MSE = \frac{1}{n}\textstyle\sum_{i=1}^{n}(y_i - \widehat{y_i})^2 \tag{10}$$

Where y_i is the value of the *i*-th observation and $\widehat{y}_i$ is the estimated value of the *i-th* estimation result.

RESULT AND DISCUSSION

1. Data

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This study consist of five independent variables ($2X_1, 3X_2$, $1.5X_2$, $2X_4$ and $0.5X_5$) and one dependent variable (Y). Where the independent variables (X_1, X_2 ,X_3, X_4 and X_5)) are stochastic. The independent variables $X_1$and X_2 spread according to the exponential distribution with parameter θ, ($X_1, X_2 \sim Eksp(\theta = 1)$). The independent variables X_3 spread according to the standard Normal distribution $X_3 \sim N(0,1)$. With $e \sim \chi^2(v = 20)$. Simulated 100 data by Software R. Variables $X_4, X_5 \sim t(v = 20)$, so that the dependent variable can be written as:

$$Y = 2X_1 + 3X_2 + 1.5X_3 + 2X_4 + 0.5X_5 + e$$

2. Estimated parameter model of quantile regression method

The results of parameter $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ dan β_5 estimation using quantile regression will be presented in the following Table 1, with the quantiles - τ =0.05, 0.25, 0.50, 0.75 dan 0.95.

Table 1. Parameter Value of β for QR for each – τ

Variabel	Parameters		L.CredIntv	U. CredIntv	Confidence Interval (95%)
			$\tau = 0.05$		
Intercept	β_0	0.0148	-0.0489	0.0234	0.0723
X_1	β_1	1.9974*	1.8057	2.0071	0.2014
X_2	β_2	2.9952*	2.9952	3.0028	0.0076
X_3	β_3	1.4972*	1.4600	1.5468	0.0868
X_4	β_4	1.9950*	1.9854	2.0024	0.0170
X_5	β_5	0.5046*	0.4739	0.5075	0.0336
			$\tau = 0.25$		
Intercept	β_0	0.0638*	0.0128	0.2627	0.2499
X_1	β_1	2.0039*	1.9255	2.0865	0.1610
X_2	β_2	3.0565*	2.9098	3.1225	0.2127
X_3	β_3	1.5000*	1.3049	1.5309	0.2260
X_4	β_4	2.0202*	1.9319	2.0441	0.1122
X_5	β_5	0.5257*	0.4425	0.5589	0.1164
			$\tau = 0.5$		
Intercept	β_0	0.5572*	0.3978	0.8441	0.4463
X_1	β_1	1.9636*	1.9073	2.1119	0.2046
X_2	β_2	3.0765*	2.8029	3.4392	0.6363
X_3	β_3	1.4016*	1.1688	1.5209	0.3521
X_4	β_4	2.0784*	1.8669	2.2495	0.3826
X_5	β_5	0.4272*	0.2067	0.6575	0.4508
			$\tau = 0.75$		
Intercept	β_0	0.9814*	0.6236	2.2946	1.6710
X_1	β_1	2.0508*	1.7865	2.4954	0.7089
X_2	β_2	3.2681*	2.9332	3.5559	0.6227
X_3	β_3	1.5076*	1.3283	1.8226	0.4943
X_4	β_4	2.2309*	1.8404	2.4938	0.6534
X_5	β_5	0.3270	-0.0414	0.5322	0.5736
			$\tau = 0.95$		
Intercept	β_0	3.1015*	1.7418	4.2802	2.5384
X_1	β_1	1.6082*	1.4425	4.8792	3.4367
X_2	β_2	3.2266*	2.3092	3.3327	1.0235
X_3	β_3	1.6844*	0.5917	2.6484	2.0567
X_4	β_4	1.9809*	0.5609	2.8455	2.2846
X_5	β_5	0.5513	-0.2797	1.5837	1.8634

* Significantly at $\alpha = 0.05$

In the table above, the significant parameters in each quantile are different, this can be seen from the smallest confidence interval of each parameter in each quantile. At quantiles 0.05, 0.25 and 0.5, the variable $(X_1, X_2, X_3, X_4, and\ X_5)$ significant because not past the zero. But X_5 in quantile 0.75 and 0.95 not significant, we can see the confidence interval (CI) past zero. Significant at alpha ($\alpha = 0.05$.)

3. Estimated parameter model of Bayesian quantile regression method

This stage, the estimation of model parameters using the Bayesian quantile regression (BQR) method with 5000 iterations and 1000 *burn-ins* can be seen in the table 2 below.

Table 2. Estimated Value of β for BQR method in quantile – τ

Variabel	Parameters		L.CredIntv	U. CredIntv	Confidence Interval (95%)
			$\tau = 0.05$		
Intercept	β_0	-0.0126	-0.0885	0.0511	0.1396
X_1	β_1	1.9993*	1.9462	2.0319	0.0857
X_2	β_2	3.0054*	2.9456	3.0494	0.1038
X_3	β_3	1.5273*	1.4704	1.6077	0.1373
X_4	β_4	2.0038*	1.9596	2.0459	0.0863
X_5	β_5	0.4791*	0.4164	0.5275	0.1111
			$\tau = 0.25$		
Intercept	β_0	0.1533*	0.0144	0.3028	0.2884
X_1	β_1	1.9897*	1.8824	2.0887	0.2063
X_2	β_2	3.0165*	2.9274	3.1101	0.1827
X_3	β_3	1.5630*	1.4585	1.6640	0.2055
X_4	β_4	1.9926*	1.9140	2.0724	0.1584
X_5	β_5	0.4157*	0.3200	0.5254	0.2054
			$\tau = 0.5$		
Intercept	β_0	0.4643*	0.2072	0.7296	0.5224
X_1	β_1	1.9793*	1.8244	2.1455	0.3211
X_2	β_2	3.0188*	2.8753	3.2008	0.3255
X_3	β_3	1.5591*	1.3927	1.7311	0.3384
X_4	β_4	1.9632*	1.8271	2.0981	0.2710
X_5	β_5	0.4439*	0.2970	0.5985	0.3015
			$\tau = 0.75$		
Intercept	β_0	1.6645*	1.1123	2.1739	1.0616
X_1	β_1	1.8761*	1.6129	2.1551	0.5422
X_2	β_2	2.9031*	2.6919	3.1926	0.5007
X_3	β_3	1.6272*	1.2654	1.9789	0.7135
X_4	β_4	1.8294*	1.4907	2.1689	0.6782
X_5	β_5	0.7068*	0.4263	1.0245	0.5982
			$\tau = 0.95$		
Intercept	β_0	2.8614*	2.3466	3.3776	1.0310
X_1	β_1	1.8934*	1.6544	2.3141	0.6597
X_2	β_2	3.1141*	2.7793	3.4560	0.6767
X_3	β_3	1.7157*	1.2327	2.4272	1.1945
X_4	β_4	1.3001*	0.9185	1.7440	0.8255
X_5	β_5	0.7603*	0.4558	1.0473	0.5915

* Significantly at $\alpha = 0.05$

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In the Table 2 above, variable $X_1, X_2, X_3, X_4, and\ X_5$ significant in all because not past the zero. Significant at alpha ($\alpha = 0.05$). Then we will compare the model QR and BQR with see the smallest MSE. Tabel 3 shows Quantile regression method and Bayesian quantile regression method heve value MSE.

Table 3. MSE Value
Quantil Regression (QR) and Bayesian Quantil Regression (BQR)

Quantile	**MSE**	
	QR	**BQR**
0.05	3.5121	2.4459
0.25	3.2907	2.1519
0.5	2.7026	**1.7445**
0.75	**2.6142**	1.7549
0.95	6.1765	5.7346

Table 3 informs to us, the Bayesian quantile regression method (BQR) has the smallest MSE value compared to quantile regression (QR), namely at quantile 0.5 in the BQR method of 1.7445. So that the best model chosen is:

$$\hat{y} = 0.4643 + 1.9793X_1 + 3.0188X_2 + 1.5591X_3 + 1.9632X_4 + 0.4439X_5$$

Next, evaluate the convergence and normality of each parameter of the model at quantiles that have the smallest MSE value of the BQR method. The convergence of parameters can be seen from the trace plot and density plot. The results of the trace plot can be seen in the figure below, namely at quantile 0.5.

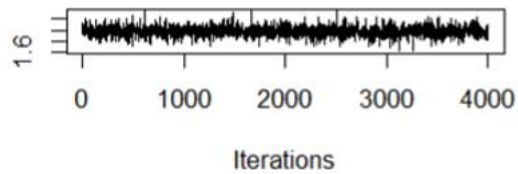

Figure 1. Trace Plot of parameter β_1 for τ = 0.5

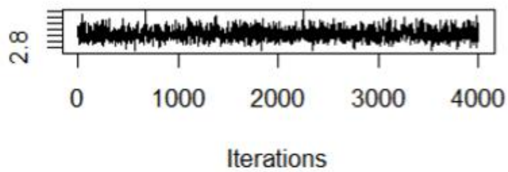

Figure 2. Trace Plot of parameter β_2 for τ = 0.5

Figure 1 and **Figure 2** shows the parameter is within the upper and lower bounds. Density plot has been normal distribution can be seen in the figure below:

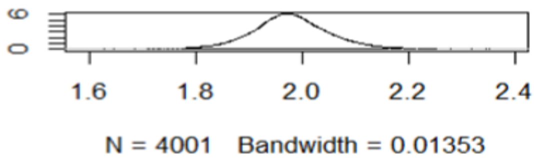

Figure 3. Density Plot of parameter β_1 for τ = 0.5

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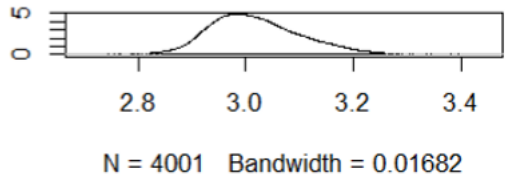

Figure 4. Density Plot of parameter β_2 for τ = 0.5

Figure 3 and **Figure 4** shows convergence to normal distributed curve. Parameters in quantil 0.5 is convergent. From figure **1, 2,3,** and **4** trace plot and density plot give model has satisfied the criterion of convergence to normal distribution.

CONCLUSION AND SUGGESTIONS

The results obtained by the Bayesian quantile regression method are superior to the indicator used MSE with the result of 1.7445. The smallest MSE value is obtained in the model that is in quantile 0.5.

$$\hat{y} = 0.4643 + 1.9793X_1 + 3.0188X_2 + 1.5591X_3 + 1.9632X_4 + 0.4439X_5$$

The result informs us that selected parameters model in quantil 0.5 is convergent and trace plot, density plot give model has satisfied the criterion of convergence to normal distribution.

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