Volume 8, Nomor 2, 2024
Numerical: Jurnal Matematika dan Pendidikan Matematika
http://journal.iaimnumetrolampung.ac.id/index.php/numerical
DOI:https://doi.org/10.25217/numerical.v8i2.

Algebraic Reasoning in Controversial Problem-Solving: Multiple Representations

Muniri[1], Musrikah[1]

[1] Universtias Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Indonesia
Correspondence: muniri@uinsatu.ac.id

Article Info

Article History:
Received: 07-10-2024
Revised: 05-11-2024
Accepted: 17-11-2024

Keywords:
Controversy;
Multiple Representations;
Reasoning

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Abstract

Controversial issues have the potential to stimulate differences or cognitive conflicts in problem-solving, both in terms of approach, argumentation, and the solutions produced. Resolving cognitive conflict can be done not only through conventional logical thinking but also by understanding objects that encompass facts, concepts, principles, and skills, as well as considering external aspects surrounding the problem. This study aims to explore the controversy reasoning problems of university students in solving multiple representation problems. This research is a case study with a qualitative approach. The selected subjects were students who experienced controversy in solving multiple representation problems. The instruments used were tests and interviews. Data analysis was carried out through data reduction, data presentation, and conclusion drawing. There are three levels of controversial reasoning: clarification level, exploration level, and initial level. Among 99 students solving multiple representation problems, the results showed that 19 students (19.19%) were at the initial level, 45 students (45.45%) at the exploration level, and 35 students (35.35%) at the clarification level. Qualitatively, controversial reasoning at the clarification level is the best compared to the exploration and initial levels. The advantage of subjects at the clarification level is that, besides understanding the problem, their algorithmic skills are highly structured. These subjects are also able to provide concise, logical arguments and find the correct solution.

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INTRODUCTION

Controversial issues have the potential to trigger cognitive conflict in individuals. The emergence of such conflicts encourages individuals to think more critically to strengthen their arguments or change their perspectives. Controversial problems arise due to incomplete understanding or lack of knowledge, which can lead to various errors in problem-solving [1]. In this context, an answer that appears correct may actually be incorrect [2]. In line with Ainsworth's opinion [3], solving controversial problems requires adequate and comprehensive understanding to be resolved correctly [4], [1]. From the above explanation, a deep understanding of

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concepts and logical thinking skills are crucial in problem-solving to reach accurate conclusions.

Several studies have been conducted on controversial problems, but there are still opportunities to expand on the findings. In mathematics, controversial problems tend to trigger cognitive conflicts in students, requiring a specific reasoning framework known as controversial reasoning. Previous research suggests that when students encounter two or more conflicting concepts, cognitive conflict emerges in their thinking [1]. This conflict can be resolved by applying proper logical thinking, understanding the problem context, grasping standard concepts and procedures, and considering external factors surrounding the problem [1], [4], [2], [5]. Another study identified three levels of controversial reasoning: initial level, exploration level, and clarification level [6]. This study focused on the stages of controversial reasoning in validating algorithmic proofs or mapping reasoning levels in mathematical problem-solving. However, difficulties or cognitive conflicts experienced in controversial reasoning when solving multiple representation problems have not been thoroughly explored.

This study aims to explore the issues related to students' controversial reasoning in solving multiple representation problems based on their reasoning levels. The level of controversial reasoning significantly influences the ability to solve multiple representation problems. The higher the level of controversial reasoning, the better an individual can solve problems [6]. In this study, multiple representation tasks involve several questions with the same substance but presented in different representations. Previous research has emphasized that such tasks are uncommon and can track students' level of understanding and their confidence in making decisions when representing ideas [7].

This study reveals that students at the initial reasoning level struggle to understand the relationships between topics or problems and often fail to choose the appropriate method or strategy, making it difficult for them to find a solution. At the exploration level, students have a good understanding of the problem and can select appropriate methods with reasonable arguments, but they still face difficulties in finding a final solution. Meanwhile, students at the clarification level demonstrate the best ability in solving multiple representation problems. They can identify relational information and easily determine various strategies or methods for problem-solving while providing logical arguments.

METHODS

This study aims to explore the level of students' controversial reasoning in solving multiple representation problems. A total of 99 participants took part in this study, all of whom were students from the Mathematics Education Program at the State Islamic University of Sayyid Ali Rahmatullah Tulungagung. They were given the task of solving two multiple representation problems. The selection of research

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subjects was conducted by examining participants' answers. The students' responses provided an illustration of the thought processes involved in solving the problems and indicated the level of controversial reasoning they possessed. Four students were then selected as research subjects, representing each level of controversial reasoning. Students with less analytical thinking skills did not realize that the two problems had the same solution.

This study employs a qualitative descriptive-exploratory approach [8], this type of research seeks to generate new ideas about a particular phenomenon and formulate the problem in detail. The researcher serves as the primary instrument, while supporting instruments include test questions and interview guidelines. The test consists of two multiple representation controversy problems, meaning both problems are substantively the same but presented in different forms. The first problem allows for controversy when solved using the inverse matrix method, as it results in a determinant or inverse value of zero. This causes the subject to experience cognitive conflict due to the controversy found. Similarly, the second problem, when solved using the elimination or substitution method, yields a controversial result because the variable values are zero.

The data sources used in this study include the subjects' answers to multiple representation problems, which represent their cognitive processes or reasoning. The researcher also used other sources, such as interview transcripts obtained from each subject based on their level of controversial reasoning, as well as information from journal articles and relevant textbooks. The problems used to explore the subjects' controversial reasoning are as follows.

First problem:

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

Determine the values of x and y that satisfy the given matrix solution!

Second problem:
Determine the values of x and y that satisfy the following system of equations!

$$2x + 3y = 10$$
$$4x + 6y = 20$$

The data collection process in this study involved the following stages: (a) designing multiple representation problems, (b) developing interview guidelines, (c) validating the multiple representation problems and interview guidelines, (d) presenting the problems to students through a test using the think-aloud method, (e) classifying responses based on the level of controversial reasoning experienced by students, and (f) conducting interviews with selected subjects based on their level of controversial reasoning.

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Data analysis was carried out both during and after data collection. This analysis aimed to achieve the research objectives. In this study, data analysis followed the approach developed by Miles and Huberman, beginning with data reduction, which focused on students' answers and interview results, standardized according to indicators of controversial reasoning in solving multiple representation problems. Next, data display was conducted by presenting narratives for each level of controversial reasoning in solving multiple representation problems. Finally, conclusion drawing/verification involved explaining the characteristics of each level of controversial reasoning in solving multiple representation problems.

RESULTS AND DISCUSSION

Result

This study involved 99 students as subjects in solving multiple representation problems. The results showed a diverse distribution across different levels: 19 students (19.19%) were at the Initial Level, 45 students (45.45%) were at the Exploration Level, and 35 students (35.35%) were at the Clarification Level. Based on the collected data, the Clarification Level had the highest number of students, followed by the Exploration Level, while the Initial Level had the fewest participants (see Table 1).

Table 1. Codification of Controversial Reasoning Levels

No	Controversial Reasoning Level	N = 99	%	Codification
1	Initial	19	19.19	SI
2	Exploration	45	45.45	SE
3	Simple Clarification	35	35.35	SK

A description of students' controversial reasoning in solving multiple representation problems is explained in the following sections. Each level is represented by one subject with a specific code. The Initial Level subject is coded as SI, the Exploration Level subject as SE, and the Clarification Level subject as SK.

1. Issues in Controversial Reasoning at the Initial Level (SI) in Solving Multiple Representation Problems

The controversial reasoning representation at the Initial Level is represented by subject SI, who struggles to properly align their ideas to find the correct answer. This is demonstrated by subject SI's written responses to Problem 1 and Problem 2, as shown in Figure 1 below.

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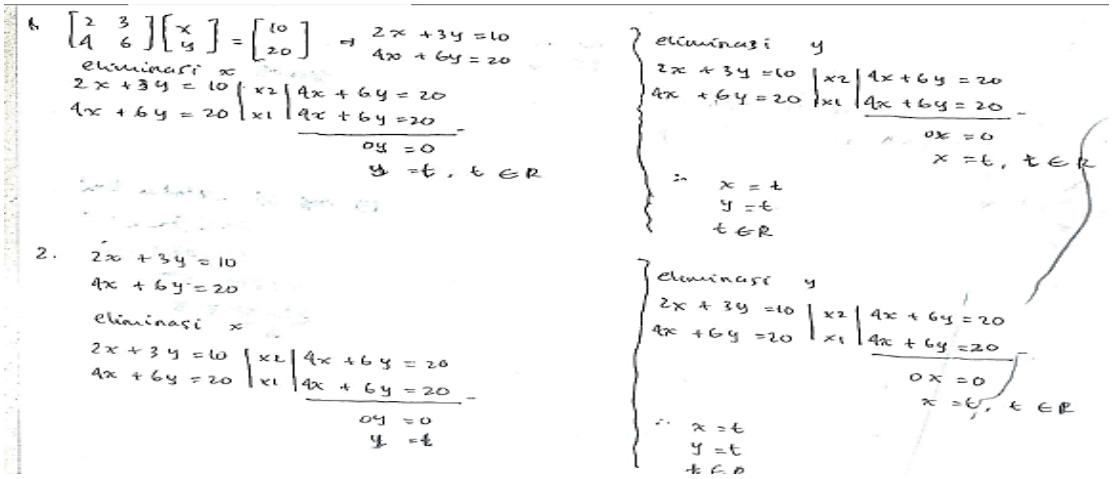

Figure 4.1 Written Response Representation of Subject SI's Problem-Solving

Based on the written response representation, subject SI immediately attempted to solve the problem using the elimination method without providing any justification. Subject SI experienced confusion when analyzing the results for both Problem 1 and Problem 2, as they wrote $0y = 0$ and $y = t$, and even stated the solution as $x = t$, $y = t$, and $t \in \mathbb{R}$, without explicitly writing the solution set.These findings are supported by the interview results with subject SI, as follows:

SI (1) **:** *I read the problem several times, then I converted it into an equation and applied elimination.*
SI (2) **:** *I used the elimination method for both Problem 1 and Problem 2. However, I couldn't find the answer because all variables disappeared on the left-hand side, and the result on the right-hand side was zero. When I reached this point, I became confused and was unable to proceed. In Problem 2, I also used the elimination method, but I couldn't find the answer either, as the result was 0 = 0.*
SI (3) : *Since the method I usually use is elimination, I didn't consider using any other method.*
SI (4) : *Wait, let me see... Oh, I just realized that both problems are actually the same.*
SI (5) : *Yes, I was confused when setting $x = t$, $y = t$, and $t \in \mathbb{R}$.*

Subject SI attempted to understand the problem by reading it repeatedly and converting the matrix form of the question into equations. Next, the subject chose the elimination method to determine the solution. However, SI encountered difficulties in reaching the final answer when all the variables disappeared or resulted in zero. SI then stopped and tried to reanalyze the problem but still failed. Eventually, subject SI concluded that the final answer was $x = t$, $y = t$, and $t \in \mathbb{R}$. The same issue occurred when solving Problem 2, which was presented in equation form. Again, using the elimination method, SI faced a deadlock where all variables were eliminated or resulted in zero.

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Thus, a subject at the initial level of controversial reasoning only mastered the elimination method. SI became confused and had no alternative ideas or strategies to determine the solution.

From the data above, it is evident that the controversial reasoning of a subject at the initial level allows them to understand the problem, even though they fail to recognize the relationship between Problem 1 and Problem 2. The subject at this level only identified one problem-solving method—the elimination method—and either forgot or failed to consider other methods, such as substitution, function graphing, or matrix inversion methods. Additionally, the subject at the initial level was unable to provide reasoning or justification for their approach, preventing them from arriving at the expected answer.

2. Issues in Controversial Reasoning at the Exploratory Level (SE) in Solving Multiple Representation Problems

Subjects at this level demonstrate reasoning skills in solving the two given problems by converting the matrix representation of the problem into a system of linear equations and then comparing them. Thus, subject SE chose the elimination method as a tool to find the solution. The written response of SE is presented in Figure 2.

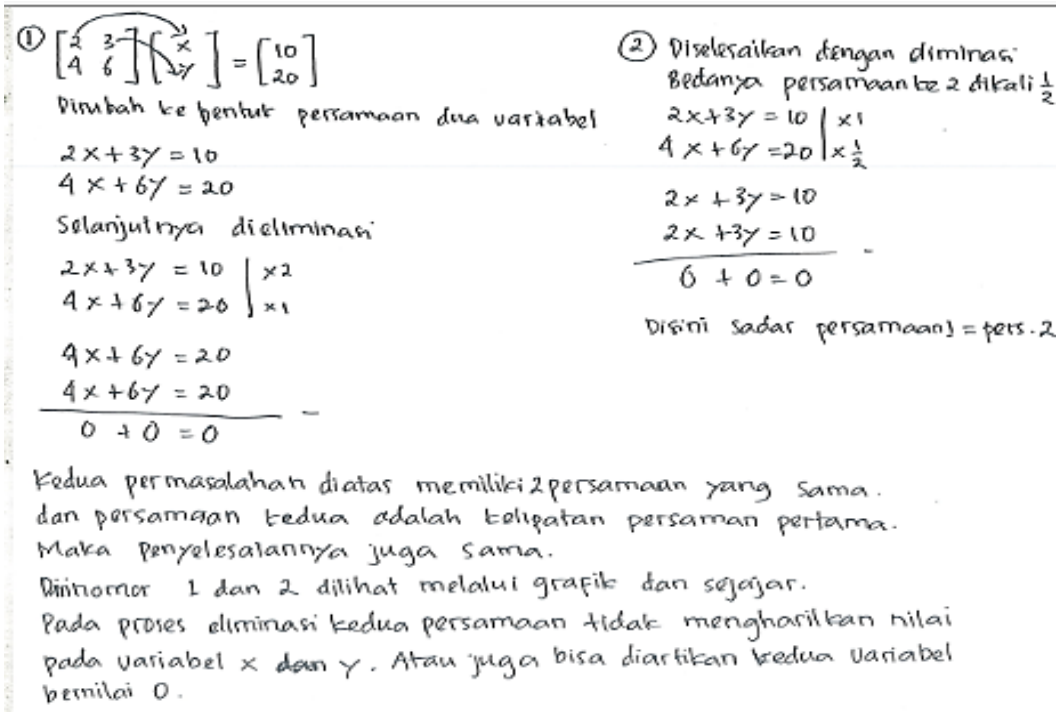

Figure 4.2: Written Representation of Problem-Solving by Subject SE

Based on the figure above, subject SE demonstrated the ability to represent and solve the two given problems by transforming the matrix form into a system of two-variable linear equations and solving them using the elimination method. After

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further analyzing the model, SE recognized that "both problems above represent the same two equations." In writing, SE stated that "the second equation is a multiple of the first equation," which implies that the solutions are the same. When viewed in terms of function graphs, this means that "the lines are parallel."

SE believed that "the resulting variables have a value of 0." As a result, SE did not continue the algebraic operations but instead provided a justification, stating that "both equations are the same, with one being a multiple of the other, so their solutions must also be the same." Additionally, SE explained that "when viewed graphically, the two equations are parallel," concluding that "there is no solution, and the values of x and y are 0."

Thus, SE demonstrated a good understanding of the problem. However, SE failed to differentiate between two parallel lines and coincident lines, which affected their ability to determine the correct final solution. The correct interpretation is that the system of equations has infinitely many solutions rather than no solution.

These findings are supported by the interview results with subject SE, as follows:

SE (1) : *"I saw that question 1 was in matrix form, so I converted it into an equation. Since question 2 was already in equation form, I proceeded directly to solving it."*

SE (2) : *"I solved it as usual using the elimination method, but I got 0 = 0 as the result. Since I encountered difficulties, I stopped and moved on to question 2. However, I got the same result, 0 = 0, and all the variables were eliminated. Because both results were identical, I decided to recheck the problem and realized that the two questions were actually the same. When I discovered this, I thought that the answers to both questions must also be the same. However, I felt that the answer I had written might not be correct. So, I rechecked question 1 and found that the first equation in question 1 was simply a multiple of the second equation in the same problem. At that point, I was unable to proceed further in solving the question."*

SE (3) : *"Because I am not very familiar with other methods."*

Based on the interview results above, it is evident that subject SE has a good understanding of the given problem. SE also realizes that question 1 and question 2 are essentially the same problem, presented in different forms—question 1 in matrix form and question 2 in equation form. SE then determines a method to solve the problem using the elimination method.

However, SE appears visibly uneasy when encountering the result 0 = 0. In this situation, SE reviews their solution by re-examining the algorithmic steps taken and realizes that the coefficients of the equations are multiples of each other, indicating that the equations represent either overlapping or parallel lines. Despite this realization, SE still fails to determine the correct solution.

From both the written responses and interview results, it can be concluded that a subject at the exploratory level of controversial reasoning is capable of understanding the problem well and identifying connections between different topics

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by repeatedly analyzing the questions. The subject is also able to find a method for solving the problem. However, SE is unable to differentiate between overlapping and parallel lines and, as a result, fails to determine the correct solution to the given problem.

3. Problematics of Controversial Reasoning at the Classification Level (SK) in Solving Multiple Representation Problems

The reasoning representation of subject SK indicates that they are able to recognize that the given problems are identical, though presented in different representations. Consequently, subject SK successfully finds a solution to the problem. The written response provided by subject SK is shown in Figure 3 below.

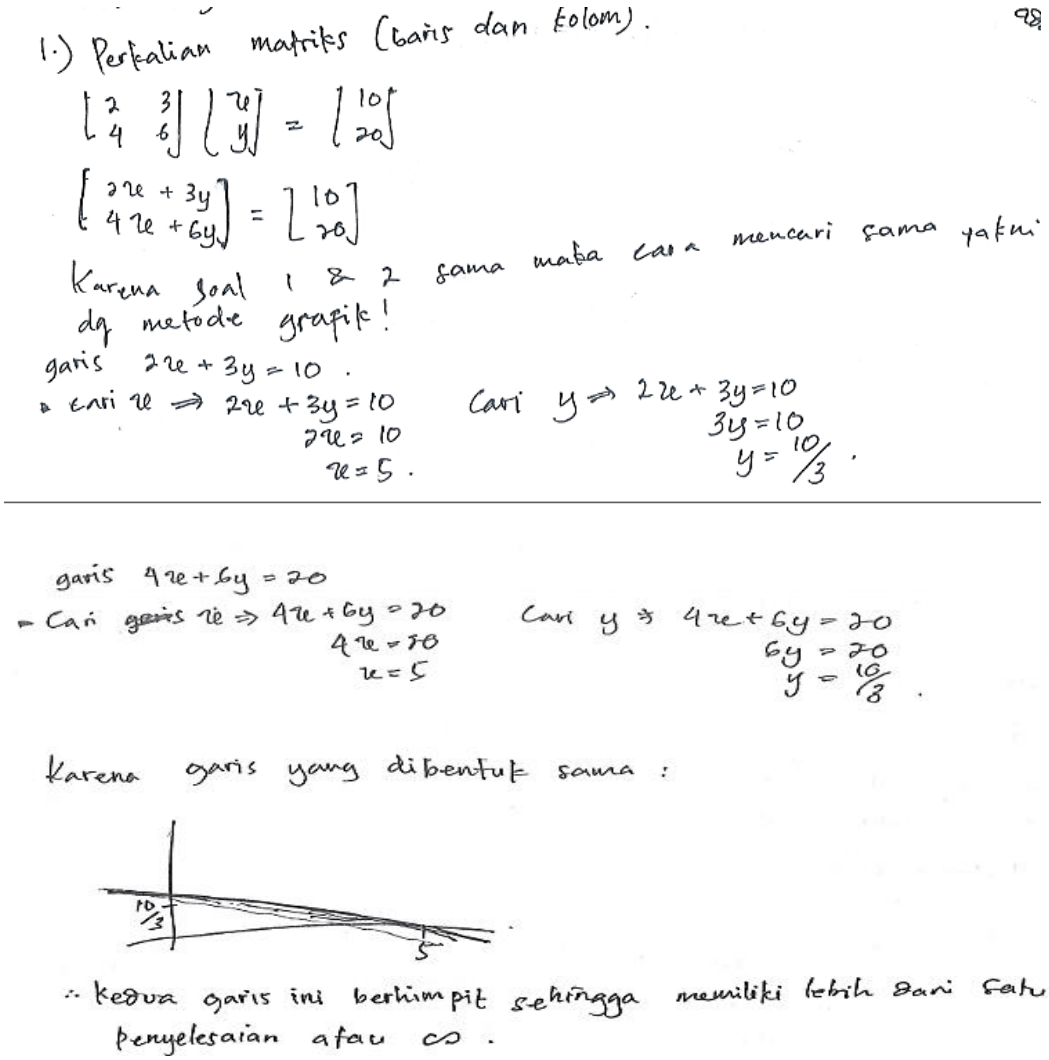

Figure 3. Representation of Written Problem-Solving by Subject SK

Based on the representation above, subject SK demonstrates an understanding of both problem 1 and problem 2 using a single solution. Initially, SK attempts an

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algorithmic approach using matrices, obtaining a new matrix form and then transforming the matrix problem in question 1 into a system of linear equations.

Next, SK tries an alternative approach using the graphical method, determining the intercepts with the axes and obtaining two coordinate pairs: (0,10/3) and (5,0). SK concludes that both equations represent the same line and therefore draws them as a single line, stating that the solution consists of infinitely many solutions.

This indicates that a subject at the classification level of controversial reasoning is capable of understanding the problem well. SK realizes that both systems of equations overlap and form a single line. Additionally, SK is able to explain that the system has infinitely many solutions, even if this explanation is only given verbally.

These findings are reinforced by the following interview with subject SK:

SK(1) : *In question 1, I tried converting the matrix form into a system of linear equations. Then, I applied the elimination method.*

SK(2) : *After reading the problem multiple times, I realized that both problems are the same. Since questions 1 and 2 are identical, I decided to give a single answer for both. Initially, I eliminated x, but then I got 0 = 0, meaning all variables disappeared. I paused for a moment, then checked the problem again. It turned out that both equations were the same, so there must be infinitely many solutions. To confirm this, I represented the equations in graphical form.*

SK(3) : *Since questions 1 and 2 were identical, I answered them with just one solution.*

SK(4) : *How is this possible? Could I be wrong? No, I'm sure! (scratches head while thinking).*

Based on the interview results above, it appears that subject SK initially did not realize that the two given problems were the same. However, after reading them multiple times and attempting to transform the representations of the given problems, SK discovered that they were actually identical in substance. Consequently, SK decided to use the elimination method to find the solution.

SK only became aware that the two problems were identical after converting the matrix form in question 1 into a two-variable linear equation. Subsequently, SK experienced confusion when the algorithmic process resulted in 0 = 0. SK found this outcome strange. Following this, SK compared the results obtained from solving question 1 and question 2 and discovered that both equations were identical. This meant that the system had infinitely many solutions, as the set of all pairs (x, y) satisfying the equation was infinite.

The written response and interview results indicate that a subject at the classification level of controversial reasoning has the ability to quickly and accurately establish relationships between pieces of information, making it easier to understand the given problem. SK realized that question 1 and question 2 were identical in substance, even though they were presented in different representations.

At this classification level, SK was able to determine an appropriate solution method, utilizing various techniques, including the elimination method, substitution

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method, and graphical method. However, upon obtaining the result 0 = 0, SK attempted to verify the solution using the graphical method, ultimately reaching the expected correct conclusion.

Discussion

Research examining controversial reasoning in solving multiple representation problems has identified three levels of controversy, each with its own indicators: the initial level, exploration level, and clarification level. A more detailed summary can be found in Table 2 below.

Table 2. Summary of Subject's Controversial Reasoning Levels in Solving Multiple Representation Problems

Controversial Reasoning Level	Indicator	Summary of Findings
Initial Level (SI)	Understands contradictions but cannot identify the components causing the contradiction and fails to obtain the correct answer.	Understands that two linear equations with two variables are presented, attempts to solve them using elimination or substitution but cannot proceed further and fails to obtain an answer.
Exploration Level (SE)	Understands contradictions and can trace the problematic components causing the contradiction but is unable to find the correct solution.	Recognizes that two linear equations with two variables are presented, identifies appropriate solution methods (elimination or substitution), and determines the cause of the contradiction but cannot arrive at the correct solution.
Clarification Level (SK)	Understands contradictions, can trace the problematic components, determines the appropriate method, provides correct answers based on concepts, and solves the problem using various aspects and logical reasoning.	Recognizes that the two given linear equations are the same despite being presented in different forms, finds a solution strategy (elimination, substitution, or graphing), determines the cause of the contradiction, and provides a correct answer with valid arguments.

In line with these findings, three levels of controversial reasoning issues can be explained as follows:

Controversial reasoning plays a crucial role in constructing mathematical knowledge and understanding. Weak conceptual understanding of mathematics triggers controversial reasoning in solving mathematical problems. The diversity of problems and the methods or strategies used to solve them influence the variation in controversial reasoning levels. At the initial level, the subject encounters problems but cannot identify their causes and fails to solve them correctly [1]. The cause of this

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controversial reasoning, as stated by Rosyadi, lies in the subject's lack of analytical and self-correction abilities [4]. The higher a subject's level of controversial reasoning, the better their problem-solving skills in mathematics [3]. Conversely, the lower the understanding of multiple representation problems, the harder it is to find the expected solution. The discussion above shows that at the initial level, the subject has a fair understanding of the problem but struggles to connect different topics or problems. They focus only on one method, elimination, and thus fail to reach the expected solution.

At the exploration level, the subject understands mathematical concepts and can establish relationships between different topics but still struggles to find the correct answer. As stated in [1], at this level, the subject has a good understanding of the problem and knows the appropriate strategies to solve it but still has difficulties in arriving at a solution. According to [2], the main cause of this controversial reasoning is the subject's limited analytical and argumentative skills. In other words, they have not yet established specific connections between components or concepts.

The higher a subject's controversial reasoning level, the better their ability to solve mathematical problems, including multiple representation problems [3]. The above discussion indicates that the exploration level is more advanced than the initial level; the subject understands the problem well, selects the appropriate method, and provides reasonable arguments. However, in this case, the subject still struggles to distinguish between coinciding and parallel lines, making it difficult to find the expected solution.

Subjects at the clarification level of controversial reasoning exhibit the highest level of reasoning. At this level, they have a solid understanding of mathematical concepts, can establish relationships between topics, find the correct answer, and provide valid arguments. As stated in [1], at the clarification level, subjects understand the problem well, identify the appropriate strategy, and can control their thought processes to find a solution. At this level, the subject encounters little to no difficulty. According to [2], this level of reasoning is supported by strong analytical, argumentative, and self-correction skills. This means that the better the subject's controversial reasoning level, the better their ability to solve mathematical problems, including multiple representation problems [3]. The discussion above suggests that subjects at the clarification level can quickly and accurately establish relationships between different pieces of information, making it easier for them to understand the problem. They can identify multiple solution strategies, including elimination, substitution, and graphing, which they consider the most effective methods for solving the given equations.

These three levels of controversial reasoning highlight the necessity for teachers to develop strategies or treatments to enhance students' reasoning abilities in overcoming difficulties in solving multiple representation problems. Understanding the findings of this study allows educators to conduct case studies in their

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classrooms, which can lead to the development of instructional methods tailored to students' reasoning levels.

Controversial issues are problems that have the potential to stimulate differences due to differing perspectives from the usual conditions [1], [2]. Controversial issues arise due to incomplete understanding, leading to conflicts within individuals, which in turn prompts them to think more critically to strengthen, support, or modify their opinions [2]. These issues allow for multiple solutions and require learners to have adequate and comprehensive understanding to be resolved appropriately. In controversial issues, seemingly correct answers may actually be incorrect, necessitating logical thinking and caution in presenting conclusions. Therefore, a strong grasp of both the subject matter and the surrounding context is essential.

Controversial issues tend to trigger cognitive conflicts in learners [9]. Learners encounter contradictions, leading to internal conflicts in their minds. Resolving these cognitive conflicts involves applying proper logical reasoning, understanding the problem context, comprehending standard concepts and procedures, and considering external factors influencing the issue. Thus, solving controversial problems requires sound reasoning, known as controversial reasoning.

Multiple representation is the ability to utilize various representations in problem-solving [3]. This includes (1) verbal mathematical representation, (2) pictorial representation (images or diagrams), and (3) graphical representation for analyzing concepts related to other concepts or variables [10]. Multiple representation, also referred to as external representation, provides unique benefits when learning complex new ideas. It can be understood by considering three essential aspects: design, function, and task [11]. According to [12], two characteristics of mathematical understanding in multiple representation problem-solving are flexibility and compartmentalization. Multiple representation fosters connections between mathematical concepts and various representational structures [13]. As such, representation is closely linked to conceptual understanding, enabling mathematics learners to solve mathematical problems accurately.

Multiple representation also plays a role in building cognitive processes, as it helps minimize misinterpretations during analysis. Additionally, it serves as an effective psychometric tool for measuring creative thinking abilities [14]. Similarly, multiple representation indirectly aids in understanding and deepening the studied concepts [15]. In some cases, multiple representation may trigger pro-contra discussions among learners, influenced by their diverse educational backgrounds. When solving mathematical word problems, solutions may involve verbal, symbolic, numeric, and graphical representations. Differences in approaches may lead to differing opinions, even controversy, among learners based on their previous learning experiences [16]. Consequently, the cognitive abilities of individual students influence problem-solving through multiple representation.

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Mathematics is often regarded as the mother of science, playing a fundamental role in the advancement of scientific knowledge and technology [17]. Mathematical learning serves as a benchmark for intellectual capacity by enhancing critical thinking and reasoning skills [18]. Mathematical understanding consists of three aspects: (1) product, which includes conceptual and principle comprehension, (2) process, which involves methods or strategies for problem-solving, and (3) attitude, which covers beliefs, opinions, and values that should be upheld [19]. The success of mathematics learning is not only measured by problem-solving abilities but also by the reasoning stages used to tackle real-life problems [20].

Mathematical problem-solving ability can be assessed through the stages of controversial reasoning. According to [1], controversial reasoning involves five stages: (1) identification—stating the core problem, (2) definition—listing facts that constrain the problem, (3) enumeration—proposing reasonable answer choices, (4) analysis—selecting the appropriate answer with logical justification, and (5) self-correction—reviewing the final answer comprehensively. Similarly, [2] categorizes controversial reasoning into three levels: (1) initial level—the ability to recognize contradictions but inability to understand their causes or reach the correct answer, (2) exploration level—the ability to identify contradictions and trace their causes, though still unable to generate correct answers, and (3) clarification level—the ability to develop logical solutions and correct answers.

CONCLUSION

This study found that understanding mathematical problems requires not only conceptual comprehension but also an understanding of the objects of study in mathematics. These objects include facts, concepts, principles, and skills. Understanding these mathematical objects plays a crucial role in problem-solving, which involves both skills and reasoning. Mapping one's level of reasoning helps in solving mathematical problems or other issues. Knowing an individual's reasoning level can be used not only to determine strategic steps or a framework for decision-making but also to align solutions with their needs.

These findings reinforce the importance of carefully examining mathematical reasoning levels. The levels of controversial reasoning, from highest to lowest, are clarification level, exploration level, and initial level. The problematics of controversial reasoning in solving multiple representation problems at the clarification level indicate that the algorithmic flow used is highly structured and correct, with brief, logical, and accurate arguments. At the exploration level, the algorithmic flow is less structured but accompanied by complete arguments, while at the initial level, the algorithmic flow is unstructured and lacks accurate arguments.

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Despite its strengths in affirming the importance of mathematical reasoning levels, this study has limitations in sample selection, lack of attention to gender, data analysis techniques, and research location. For instance, the sample in this study consists of only three individuals, each representing a different level of controversial reasoning. The small sample size may result in overlooking other controversial reasoning phenomena. In line with this, further research is needed to accommodate a larger sample size, gender considerations, advanced data analysis techniques, or broader research coverage to obtain more comprehensive answers to the identified problems.

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Print ISSN: 2580-3573, Online ISSN: 2580-2437